Bremsstrahlung In High Density Mediums: Extended Information for "Asymmetric Backward Peaking Radiation Pattern from a Relativistic Particle Accelerated by Lightning Leader Tip Electric Field"

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Introduction Bremsstrahlung radiation patterns were predicted to be forward and backward peaking with associated novel bremsstrahlung asymmetry, R (Yücemöz & Füllekrug, 2021). In addition, the time evolution of dipole radiation pattern into forward-backward peaking was demonstrated. The reasoning for the existence of forward-backward peaking due to the collapse and separation of the lobes of the dipole radiation pattern was associated with the conservation of symmetry axes (Yücemöz & Füllekrug, 2021). Furthermore, this extended modelling of the bremsstrahlung process into high-density mediums, reveals a new outcome that symmetry of the bremsstrahlung asymmetry, R about the axis perpendicular to the direction of particle's motion, between forward and backward peaking

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side is broken. Increasing refractive index causes bremsstrahlung asymmetry, R to exist at different ratios between forward and backward peaking radiation lobe pairs. Increasing refractive index increases the bremsstrahlung asymmetry between the front radiation lobe pairs compared to backward radiation lobe pairs. Furthermore, increasing the refractive index shortens the radiation length in the backward peaking radiation side.

0.1. Aims & Objectives

This theoretical approach aims to extend the previous bremsstrahlung model to different high-density mediums where particle speed can exceed the speed of an emitted electromagnetic wave. In addition, to predict and understand how radiation patterns and previously introduced bremsstrahlung asymmetry parameter R changes within these high refractive index mediums. To achieve the stated aim, particle emitted frequency-dependent refractive index, $n(\omega)$ is identified, expressed in terms of Solid angle, $\Omega_{n,\beta}$, (Eq. 12) and substituted into the main radiation pattern equation (Eq. 13).

1. Supplemental Text, Equations and Figure S1

1.1. Equation Relating Top and Bottom Radiation lobes Together Using Bremsstrahlung Asymmetry, R

Considering forward peaking part of the overall radiation pattern. Radiation intensity, I of top lobe (I_T) can be related to the bottom lobe (I_B) with bremsstrahlung asymmetry,R mathematically by

$$I_T = \frac{I_B}{(1-R)} \tag{1}$$

This information can be used to find the bremsstrahlung angle, θ_{Brem} between the two forward peaking radiation lobes that are bremsstrahlung asymmetric, with asymmetry value R.

By knowing the distance, Ξ(m) between the maximum points of two radiation intensities, I in the forward peaking lobes, bremsstrahlung angle between the two forward peaking lobes can be written as,

$$cos(\theta_{Brem}) = \frac{I_T^2 + I_B^2 - \Xi^2}{2I_TI_B} \tag{2}$$

The equation 1 and 2 can also be used for backward peaking radiation lobes.

1.2. Extending Bremsstrahlung modelling to High Density Mediums

The equation for translating the angular frequency of the emitted wave into the laboratory frame for the Doppler shift is given by (Jackson, 1999, p. 720, eq. 15.40).

$$\omega' = \gamma\omega(S_{SpecialR} - \beta S_{SpecialR}cos(\theta_{n,\beta})). \tag{3}$$

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$\theta_{n,\beta}$ is the angle between the emitted radiation unit vector and the particle velocity vector.

Previously, bremsstrahlung radiation pattern with bremsstrahlung asymmetry, R is given by (Yücemöz & Füllekrug, 2021)

$$\frac{d^2 I}{d\omega\Omega_{rad}} = \frac{z^2 e^2 (\gamma\omega(S_{SpecialR} - \beta S_{SpecialR}cos(\theta_{n,\beta}))^2}{4\pi^2 c\epsilon_0} \left| sin(\theta_{n,\beta}) \left[-\frac{s_{fv}s_f z (s_{ft})^{1.461} 4.365 \times 10^{26}}{c} \right. \right.$$
$$\left[\pi^{1/2} 2^{-(1/2)\nu_1} \alpha^{-\nu_1-1} e^{-\frac{y^2\alpha^{-2}}{8}} \times D_{\nu_1}(2^{-1/2}\alpha^{-1}y) \right] +$$
$$\left. \left. \frac{s_{fv}z s_f (s_{ft})^{1.5} 1.565 \times 10^{27}}{c} \left[\pi^{1/2} 2^{-(1/2)\nu_2} \alpha^{-\nu_2-1} e^{-\frac{y^2\alpha^{-2}}{8}} \times D_{\nu_2}(2^{-1/2}\alpha^{-1}y) \right] \right] \right|^2, \tag{4}$$

where R is the bremsstrahlung asymmetry control parameter, which plays a crucial role in distinguishing the parameter y from α. $D_v(z)$ is the parabolic cylinder function.

The parabolic cylinder function is given by (Whittaker & Watson, 1927, p. 347)

$$D_v(z) = 2^{v/2+1/4} z^{-1/2} W_{v/2+1/4,1/4}(1/2z^2), \tag{5}$$

where, $W_{v/2+1/4,1/4}(1/2z^2)$ is a Whittaker function (Whittaker & Watson, 1927, p. 346) and

$$W_{\kappa,\mu}(1/2z^2) = \frac{\Gamma(-2\mu)M_{\kappa,\mu}(1/2z^2)}{\Gamma(1/2-\mu-\kappa)} + \frac{\Gamma(2\mu)M_{\kappa,-\mu}(1/2z^2)}{\Gamma(1/2+\mu-\kappa)}, \tag{6}$$

where, $M_{\kappa,\mu}(1/2z^2)$ is another Whittaker function (Kiyosi Ito and The Mathematical Society of Japan, 1993) & (Whittaker & Watson, 1927, p. 347) and

$$M_{\kappa,\mu}(1/2z^2) = {}_1F_1(1/2+\mu-\kappa; 2\mu+1; 1/2z^2)(1/2z^2)^{1/2+\mu} e^{-1/21/2z^2}. \tag{7}$$

For the second term of equation (27) with Whittaker M function of negative μ

$$M_{\kappa,-\mu}(1/2z^2) = {}_1F_1(\mu - \kappa; 2\mu; 1/2z^2)(1/2z^2)^{1/2+\mu}e^{-1/21/2z^2}, \tag{8}$$

where ${}_1F_1(1/2 + \mu - \kappa; 2\mu + 1; z)$ is a confluent hypergeometric function of the first kind (Abramowitz & Stegun, 1972) and

$$_1F_1(1/2 + \mu - \kappa; 2\mu + 1; 1/2z^2) = \frac{U_{\kappa,\mu}(1/2z^2)}{e^{-1/2z^2/2}(1/2z^2)^{\mu+1/2}}, \tag{9}$$

where, $U_{\kappa,\mu}(\frac{z^2}{2})$ is the confluent hypergeometric function of the second kind, named Kummer's U function such that

$$U_{\kappa,\mu}\left(\frac{z^2}{2}\right) = \left(\frac{z^2}{2}\right)^{\mu+1/2}e^{-\frac{z^2}{4}}\sum_{n=0}^{\infty}\frac{(m-k+1/2)_n}{n!(2\mu+1)_n}\left(\frac{z^2}{2}\right)^n. \tag{10}$$

Where the constant quantities z, κ and μ are defined as $z = 2^{-1/2}\alpha^{-1}y$, $\kappa = v/2 + 1/4$, $\mu = 1/4$.

Transforming equation 4 into a form that allows to predict how radiation pattern would be affected in a denser medium where electromagnetic wave speed can slow down below particle speed. This can be done by dividing speed of light in vacuum by frequency dependent refractive index n(ω).

Every speed of light in vacuum "c" parameter can be replaced with "$\frac{c}{n(\omega)}$".

Therefore,

$$\frac{c}{n(\omega)} = \frac{c}{\sqrt{\epsilon_r\mu_r}} = \frac{c}{\sqrt{\left(1 - \frac{i\sigma}{\epsilon_0\omega'}\right)\mu_r}} \tag{11}$$

From equation 3, we can transform angular wave frequency from moving frame of reference, ω' to laboratory frame of reference, ω in equation 13.

$$\frac{c}{\sqrt{\left(1-\frac{i\sigma}{\epsilon_0\omega'}\right)\mu_r}} = \frac{c}{\sqrt{\left(1-\frac{i\sigma}{\epsilon_0\left[\gamma\omega(S_{SpecialR}-\beta S_{SpecialR}cos(\theta_{n,\beta}))\right]}\right)\mu_r}} \tag{12}$$

Where σ is the electrical conductivity of the medium. Hence,

$$\frac{d^2I}{d\omega\Omega_{rad}} = \frac{z^2e^2(\gamma\omega(S_{SpecialR}-\beta S_{SpecialR}cos(\theta_{n,\beta}))^2}{4\pi^2c\epsilon_0}\bigg| sin(\theta_{n,\beta})$$

$$\left[-\frac{s_{fv}s_fz(s_{ft})^{1.461}4.365\times10^{26}}{\frac{c}{\sqrt{\left(1-\frac{i\sigma}{\epsilon_0\left[\gamma\omega(S_{SpecialR}-\beta S_{SpecialR}cos(\theta_{n,\beta}))\right]}\right)\mu_r}}}\right.$$

$$\left[\pi^{1/2}2^{-(1/2)\nu_1}\alpha^{-\nu_1-1}e^{-\frac{y^2\alpha^{-2}}{8}}\times D_{\nu_1}(2^{-1/2}\alpha^{-1}y)\right]+$$

$$\left.\frac{s_{fv}zs_f(s_{ft})^{1.5}1.565\times10^{27}}{\frac{c}{\sqrt{\left(1-\frac{i\sigma}{\epsilon_0\left[\gamma\omega(S_{SpecialR}-\beta S_{SpecialR}cos(\theta_{n,\beta}))\right]}\right)\mu_r}}}\left[\pi^{1/2}2^{-(1/2)\nu_2}\alpha^{-\nu_2-1}e^{-\frac{y^2\alpha^{-2}}{8}}\times D_{\nu_2}(2^{-1/2}\alpha^{-1}y)\right]\right]\bigg|^2,$$

$$\tag{13}$$

where the Lorentz factor γ is $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ (dimensionless), and ω is the received angular frequency in the laboratory (stationary) frame of reference in rad/s. In addition, the new definition of α is $\alpha^2 = \frac{b^R(\gamma\omega(S_{SpecialR}-\beta S_{SpecialR}cos(\theta_{n,\beta})))^R(sin(\theta_{n,\beta}))^R}{(\tau^R)^2\left[\frac{c}{\sqrt{\left(1-\frac{i\sigma}{\epsilon_0\left[\gamma\omega(S_{SpecialR}-\beta S_{SpecialR}cos(\theta_{n,\beta}))\right]}\right)\mu_r}}\right]^R}$ $[s^{-2}]$, and the new definition of y is $y = \frac{\gamma\omega(S_{SpecialR}-\beta S_{SpecialR}cos(\theta_{n,\beta}))sin(\theta_{n,\beta})a}{\left[\frac{c}{\sqrt{\left(1-\frac{i\sigma}{\epsilon_0\left[\gamma\omega(S_{SpecialR}-\beta S_{SpecialR}cos(\theta_{n,\beta}))\right]}\right)\mu_r}}\right]\tau}$ $[s^{-1}]$.

1.3. Results

This section presents previous prediction of high frequency radiation pattern of a single electron with bremsstrahlung, R and Doppler frequency asymmetry accelerated under an external lightning leader tip electric field using the derived equation 4. In addition, results extend (Eq. 13) to demonstrate what would happen to radiation length and novel bremsstrahlung asymmetry, R if bremsstrahlung process takes place inside a high density medium.

A high-density medium for a single electron is chosen to be water. As can be seen in figure E1, as the bremsstrahlung electron travels inside water following a curved trajectory, a high-density medium causes larger bremsstrahlung asymmetry in the forward peaking radiation. This difference in the magnitude of bremsstrahlung asymmetry, R in forward and backward peaking radiation is a novel asymmetry in the radiation pattern. The reason for asymmetric bremsstrahlung asymmetry, R in forward and backward peaking lobes is mainly due to the propagation direction of the emitted radiation (In forward peaking, emitted radiation propagates in the direction of motion of particle. Whereas, in backward peaking lobes, radiation propagates in the opposite direction to the particle's motion.) and causes of Doppler effect on each specific radiation lobe. Each individual radiation lobe in vacuum have had different radiation energies and frequencies due to Doppler effect and bremsstrahlung asymmetry. As all radiation lobes have had different properties, when placed in a high density mediums where wave properties such as frequency and energy determines how much of an electromagnetic wave is going to be slowed down, this introduced another novel asymmetry in the novel bremsstrahlung asymmetry, R. This information is crucial and novel. High-density medium also causes backward peaking radiation length

to decrease. This is mainly because backward peaking radiation propagates in opposite direction to the particle motion meaning radiation velocity is negatively affected. Another contributing factor to decreasing radiation length in backward peaking lobes is, as the backward peaking lobes are much closer to each other (low angle of separation) and more compact than forward peaking radiation lobes, they almost align with the line of particle's velocity vector, hence more backward peaking lobes are more susceptible to the physical causes of Doppler effect.

1.4. Summary

Previously the radiation patterns of combined parallel and perpendicular motions from the accelerated relativistic particle at low and high frequencies of the bremsstrahlung process with an external lightning electric field were explained. The primary outcome was that radiation patterns have four relative maxima with two forward peaking and two backward peaking lobes. The asymmetry of the radiation pattern, i.e., the different intensities of forwarding and backward peaking lobes, is caused by the Doppler effect. A novel outcome is that bremsstrahlung has an asymmetry of the four maxima around the velocity vector caused by the curvature of the particle's trajectory as it emits radiation. This extended work reports another novel asymmetry in the overall radiation pattern. Previously stated bremsstrahlung asymmetry, R was an asymmetry in the radiation lobe pairs about particles velocity vector. Bremsstrahlung asymmetry used to occur at the same level in both forward radiation lobe pairs and backward radiation lobe pairs. However, in high-density mediums where the emitted wave can lag behind the speed of the particle, symmetry of the magnitude of bremsstrahlung asymmetry, R differs between forward peaking radiation lobe pairs relative to backward peaking radiation lobe pairs. This

is another novel asymmetry and it causes bremsstrahlung asymmetry, R to be larger in the forward peaking compared to backward peaking radiation. The outcome is the shrink in radiation length that occurs in the backward peaking lobes. This extended mathematical modeling of the bremsstrahlung process into different high-density mediums helps to better understand the physical processes of a single particle's radiation pattern, which might assist the interpretation of observations with networks of radio receivers and arrays of γ-ray detectors.

In summary, the bremsstrahlung process occurring in a high-density medium causes bremsstrahlung asymmetry, R to be a different magnitude in forward and backward peaking radiation direction. This difference in bremsstrahlung asymmetry, R is a novel asymmetry in radiation pattern. Bremsstrahlung asymmetry was found to occur at a larger magnitude in a forward direction with a larger bremsstrahlung asymmetry, R number. Larger bremsstrahlung asymmetry in forward direction is enhanced by the Doppler effect acting at different magnitudes in each lobe of the forward peaking radiation because of the larger bremsstrahlung asymmetry R between two forward peaking lobes. This is because, Doppler frequency doesn't scale linearly with the emitted radiation frequency (equation 3) and bremsstrahlung asymmetry causes the difference in emitted radiation frequency. Finally, radiation length in a high-density medium of the bremsstrahlung process was found to be shorter in the backward peaking radiation. This is firstly because of backward peaking radiation propagates in the opposite direction of motion meaning their speed is negatively affected compared to the speed of particle. Secondly, each lobe of backward peaking radiation are more susceptible to Doppler effect as the separation angle between them is smaller, they are more compact and radiating almost parallel to each

other. Smaller separation angle means wave propagation is almost in line with the parti-
cle's velocity vector. Radiation length will be shorter in backward direction and longer in
forward direction with smaller separation angle closer to the velocity vector of the particle
as this is where the causes of Doppler effect is at maximum. Top and bottom radiation
intensities are related to each other by the bremsstrahlung asymmetry parameter R given
in the equation 1.

Quite importantly, Doppler effect is at maximum when the emitted radiation is in
the direction of particle's motion, in other terms particle's velocity vector. Therefore,
if bremsstrahlung asymmetry, R exists in straight-line trajectories, this is due to two
reasons. Shift in particle's velocity vector that causes asymmetric angular distance of
each radiation lobe (See figure E2, Ω_1, Ω_2 , Ω_3 , Ω_4) from the particle's velocity vector.
Alternatively, there could be no shift in particle's velocity vector. However, each radiation
lobe, when they peak in forward-backward direction, they peak independently at different
magnitudes away from the particle's velocity vector (See figure E2, $\Omega_1 \neq \Omega_2 \neq \Omega_3 \neq$
Ω_4). Hence, Doppler effect can produce different emitted radiation lengths on relativistic
particle's forward-backward peaking radiation pattern, therefore causes bremsstrahlung
asymmetry, R in straight line trajectories.

1.5. Key Points

Different mediums create another new asymmetry and cause new asymmetric
bremsstrahlung asymmetry, R about the line perpendicular to the particle's direction
of motion (velocity vector). Higher the density of the medium, slower the EM wave
speed, higher radiation bend and higher the bremsstrahlung asymmetry, R in the forward
peaking compared to backward peaking. Higher density medium causes backward peaking

radiation length to decrease. Decrease in radiation length is higher in the backward direction as backward peaking lobes are always at lower intensity and have low energies due to Doppler effect. Effects of Doppler effect is larger on radiation lobes when the separation angle between two radiation lobes decreases. This is because, decreasing separation angle between two lobes brings two lobes closer into the line of particle's direction of motion. As the Doppler effect doesn't scale linearly with the emitted electromagnetic radiation frequency (equation 3), larger bremsstrahlung asymmetry, R causes larger difference in magnitude of Doppler effect on each forward peaking radiation lobe. In turn, resultant individual Doppler effect on each lobe enhances the increase in the bremsstrahlung asymmetry, R in the forward peaking lobes. Difference in the causes of Doppler effect is larger between forward peaking radiation lobes due to high bremsstrahlung asymmetry, R. Difference in the causes of Doppler effect is minimum between backward peaking radiation lobes due to smaller bremsstrahlung asymmetry, R. Bremsstrahlung asymmetry, R determines the magnitude of Doppler effect in each individual radiation lobe which in turn enhances the increase in difference in radiation length, and increases the bremsstrahlung asymmetry further. Finally, if bremsstrahlung asymmetry, R exists in straight line trajectories there could be shift in particle's velocity vector that causes asymmetric angular distance of each radiation lobe (See figure E2, Ω_1, Ω_2 , Ω_3 , Ω_4) from the particle's velocity vector. Alternatively, each radiation lobe, when they peak in forward-backward direction, they could peak independently at different magnitudes away from the particle's velocity vector (See figure E2, $\Omega_1 \neq \Omega_2 \neq \Omega_3 \neq \Omega_4$).

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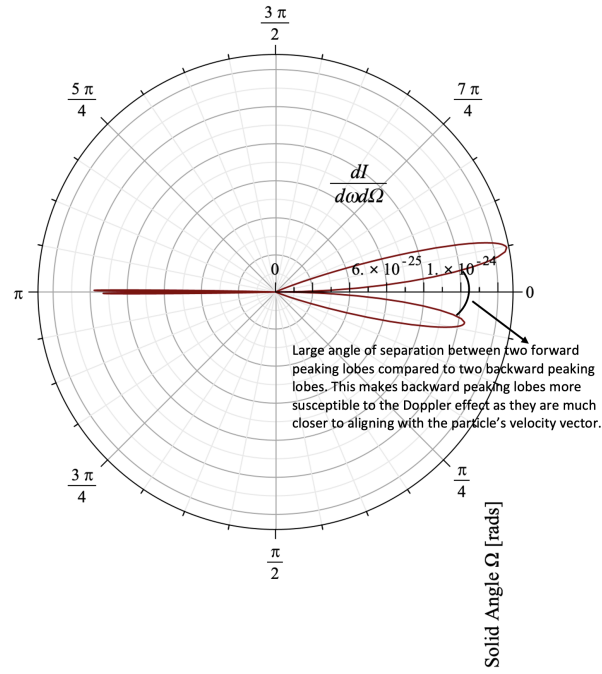
Figure E1. The Radiation patterns are emitted by the anti-clockwise rotating charged particle - bremsstrahlung process. High density (particle travelling inside the water) medium causes a novel asymmetry about a line perpendicular to the direction of motion of a particle. This novel asymmetry causes bremsstrahlung asymmetry, R to occur at different proportions in forward and backward peaking lobe pairs. Therefore, bremsstrahlung asymmetry, R is higher in forward peaking radiation and lower in backward peaking radiation. Moreover, a high-density medium also causes radiation length to shorten in backward peaking radiation. Plot is in Polar co-ordinates. Horizontal axis gives the radiation intensity per Solid angle, Ω, per emitted angular radiation frequency, ω. In addition, angle of the Polar plot is the Solid angle, Ω. The values used for plotting are: mean free time $\tau = 30$ μs, number of charges $z = 1$, $a = 100$ μm, $b = 1$ nm (a and b are related to mean free path), $s_{ft} = 1$, $s_f = 1$, $S_{SpecialR} = 1$, velocity-time scaling factor $s_{ftv} = 1 \times 10^9$ and velocity scaling factor $s_{fv} = 8.19 \times 10^{-11}$. Finally, the bremsstrahlung asymmetry is $R = 1/8$. In addition, $\frac{1}{9} \leq R \leq \frac{1}{3}$., medium conductivity, $\sigma = 0.005$, relative permeability, $\mu_r = 0.99$

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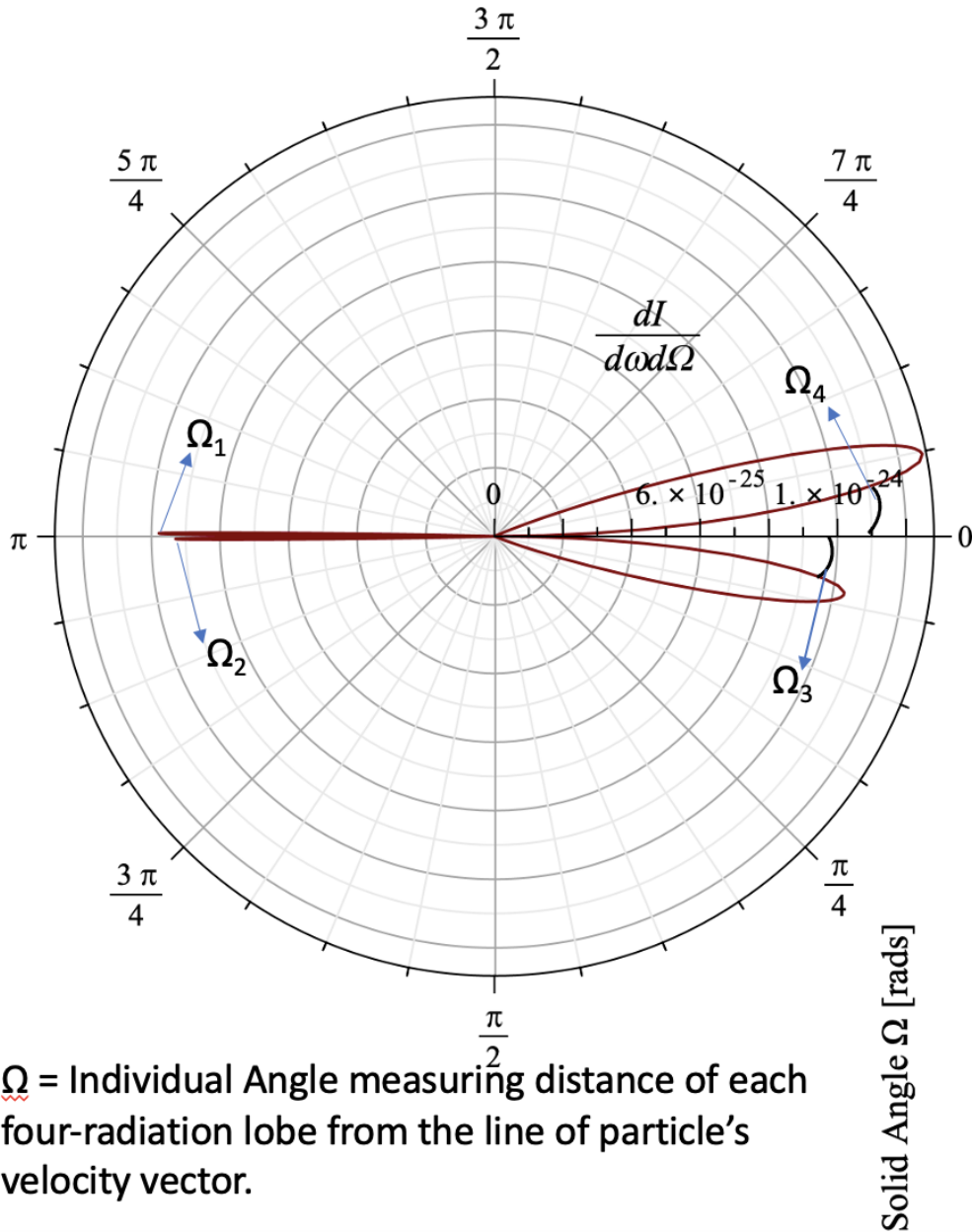
Figure E2. Shift in particle's velocity vector that causes asymmetric angular distance of each radiation lobe (See figure E2, Ω_1, Ω_2 , Ω_3 , Ω_4) from the particle's velocity vector. Alternatively, there could be no shift in particle's velocity vector. However, each radiation lobe, when they peak in forward-backward direction, they peak independently at different magnitudes away from the particle's velocity vector (See figure E2, $\Omega_1 \neq \Omega_2 \neq \Omega_3 \neq \Omega_4$). Hence, Doppler effect can produce different emitted radiation lengths on relativistic particle's forward-backward peaking radiation pattern, therefore causes bremsstrahlung asymmetry, R in straight line trajectories.

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