QE and CAD Examples

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1 CAD Examples

Most examples are pure CAD versions of those from David Wilson's original database at [Wil13], if they weren't pure CAD examples already. If not stated, this is their original source, or this repository is the original source.

1.1 ArcSin

$$-2\,xy = 0 \wedge -x^2 + y^2 < -1 \vee -16\,x^3y + 16\,xy^3 + 8\,xy = 0 \wedge 4\,x^4 - 24\,x^2y^2 + 4\,y^4 - 4\,x^2 + 4\,y^2 \leq -1 \vee y = 0 \wedge -x \leq -1 \vee y = 0 \wedge x \leq -1$$

1.2 Ellipse

$$x^2 + y^2 - 1 = 0 \vee -a^2b^2 + a^2y^2 + b^2c^2 - 2\,b^2cx + b^2x^2 = 0 \wedge 0 < a \wedge a < 1 \wedge 0 < b \wedge b < 1 \wedge 0 < c \wedge c < 1$$

Originally from [BH91]

1.3 Kahan

$$8\,x^3y + 8\,xy^3 + 84\,x^2y + 20\,y^3 + 288\,xy + 324\,y = 0 \wedge -4\,x^4 + 4\,y^4 - 52\,x^3 + 12\,xy^2 - 225\,x^2 + 63\,y^2 - 324\,x < 0 \vee 2\,y = 0 \wedge 2\,x < -9 \vee 8\,y = 0 \wedge 8\,x^2 + 8\,y^2 + 56\,x < -96 \vee y = 0 \wedge x^2 + y^2 + 7\,x < -12 \vee 8\,x^3y + 8\,xy^3 + 84\,x^2y + 20\,y^3 + 288\,xy + 324\,y = 0 \wedge -4\,x^4 + 4\,y^4 - 52\,x^3 + 12\,xy^2 - 252\,x^2 + 36\,y^2 - 540\,x \leq 432 \wedge 4\,x^4 - 4\,y^4 + 52\,x^3 - 12\,xy^2 + 225\,x^2 - 63\,y^2 + 324\,x < 0 \vee 2\,y = 0 \wedge -2\,x < 6 \wedge 2\,x \leq 0 \wedge 8\,y = 0 \wedge -8\,x^2 - 8\,y^2 - 56\,x < 96 \wedge 2\,x^2 + 2\,y^2 + 8\,x \leq 0$$

1.4 Random

$$4\,xy^2 + 16\,x^2 - 4\,z + 1 = 0 \vee 2\,y^2z + 4\,x + 1 = 0 \wedge 2\,x^2z - 2\,y^2 - x < 0$$

Originally from [BH91]

1.5 2D Example

$$x^2 + y^2 - 1 = 0 \wedge 4\,xy < 1 \vee x^2 + y^2 - 8\,x - 2\,y + 16 = 0 \wedge 4\,xy - 4\,x < 17$$

1.6 3D Example

$$x^2 + y^2 + z^2 - 1 = 0 \wedge 4\,xyz < 1 \vee x^2 + y^2 + z^2 - 8\,x - 2\,y - 4\,z + 20 = 0 \wedge 4\,xyz - 8\,xy - 4\,xz - 16\,yz + 8\,x + 32\,y + 16\,z < 33$$

1.7 3D Sphere

$$z - x < 1 \wedge z - y < 1 \wedge x^2 + y^2 - 1 = 0$$

1.8 Arnon-84

$$2\,x^4 + y^4 - 3\,x^2y - 2\,y^3 + y^2 = 0$$

Originally from [ACM84]

1.9 Arnon-84-2

$$[9\,x^4 + 96\,x^2y + 105\,x^2 + 144\,y^2 + 70\,x - 98 = 0, x^3 + xy^2 + 6\,xy + 9\,x = 0]$$

Originally from [ACM84]

1.10 Collision

$$t^2 - 2\,tx + x^2 + 4\,y^2 - 80\,y + 396 = 0 \wedge 0 < t \vee 5\,a^2t^2 - 2\,atx - 8\,aty + x^2 + 4\,y^2 - 4 = 0 \wedge 0 < a$$

Originally from [BH91]

1.11 Intersection

$$xz + yz - 2\,x = 0 \vee z^2 - y = 0 \wedge 2\,x^2 - y^2 - z^2 < 0$$

Originally from [BH91]

1.12 ProjectionCAD 1

$$x^2 + y^2 - 1 = 0 \wedge xy < 1/4 \vee (x - 4)^2 + (y - 1)^2 - 1 = 0 \wedge (x - 4)\,(y - 1) < 1/4$$

Originally from [Eng15]

1.13 ProjectionCAD 2

$$x^2 + y^2 - 1 = 0 \wedge x^3 + y^3 - 1 = 0 \wedge xy < 1/4 \vee (x - 4)^2 + (y - 1)^2 - 1 = 0 \wedge (x - 4)^3 + (y - 1)^3 - 1 = 0 \wedge (x - 4)\,(y - 1) < 1/4$$

Originally from [Eng15]

1.14 Solotareff

$3\,x^2 - a - 2\,x = 0 \wedge 0 < x^3 - ax - x^2 + a - 2\,b - 2 \wedge 0 < 4\,a - 1 \wedge 4\,a < 7 \wedge 0 < x + 1 \wedge x < 0 \vee 3\,y^2 - a - 2\,y = 0 \wedge 0 < y^3 - ay - y^2 - a + 2 \wedge 0 < 4\,b + 3 \wedge 4\,b < 3 \wedge 0 < y \wedge y < 1$

Originally from [BH91]

1.15 Concentric Circles

$$[x^2 + y^2 - 9 = 0, x^2 + y^2 - 1 = 0]$$

Originally from [Dav11]

1.16 Non-Concentric Circles

$$[x^2 + y^2 - 9 = 0, x^2 + (y-1)^2 - 1 = 0]$$

Originally from [Dav11]

1.17 Sphere and Catastrophe

$$[x^2 + y^2 + z^2 - 1 = 0, z^3 + xz + y = 0]$$

Originally from [McC88]

1.18 McCallum Trivariate Random Polynomial

$$(y-1)\,z^4 + xz^3 + x\,(1-y)\,z^2 + (y - x - 1)\,z + y = 0$$

Originally from [McC88]

1.19 Piano Movers Problem (Davenport) (Original)

$$(x-a)^2 + (y-b)^2 - 9 = 0 \wedge 0 \le yb \vee 0 \le x\,(y-b)^2 + y\,(a-x)\,(y-b) \wedge 0 \le (y-1)\,(b-1) \vee 0 \le (x+1)\,(y-b)^2 + (y-1)\,(a-x)\,(y-b) \wedge 0 \le ax \vee 0 \le y\,(x-a)^2 + x\,(b-y)\,(x-a) \wedge 0 \le (x+1)\,(a+1) \vee 0 \le (y-1)\,(x-a)^2 + (x+1)\,(b-y)\,(x-a)$$

Originally from [WDEB13]

2 QE Examples

Many of the given examples are from David Wilson's original database at [Wil13]. If not otherwise stated, this is their original source, or this repository is the original source.

3 QE Examples

3.1 Hong-90

$$(\exists a)\,(\exists b)\;\; r+s+t=0 \wedge rs+rt+st-a=0 \wedge rst-b=0$$

Originally from [Hon90]

3.2 Quartic

$$(\forall x)\;\; 0 \le x^4+px^2+qx+r$$

Originally from [Laz88]

3.3 YangXia

$$(\exists s)\,(\exists b)\,(\exists c)\;\; a^2h^2-4\,s\,(s-a)\,(s-b)\,(s-c)=0 \wedge 2\,Rh-bc=0 \wedge 2\,s-a-b-c= 0 \wedge 0<b \wedge 0<c \wedge 0<R \wedge 0<h \wedge 0<a+b-c \wedge 0<b+c-a \wedge 0<c+a-b$$

Originally from [BG06]

3.4 Collision

$$(\exists y)\,(\exists x)\,(\exists t)\;\; 1/4\,\left(x-t\right)^2+\left(y-10\right)^2-1=0 \wedge 1/4\,\left(-at+x\right)^2+\left(-at+y\right)^2-1= 0 \wedge 0<t \wedge 0<a$$

Originally from [BH91]

3.5 Cyclic-3

$$(\exists b)\,(\exists a)\;\; a+b+c=0 \wedge ab+ac+bc=0 \wedge abc-1=0$$

3.6 Cyclic-4

$$(\exists c)\,(\exists b)\,(\exists a)\;\; a+b+c+d=0 \wedge ab+ad+bc+cd=0 \wedge abc+abd+acd+bcd=0 \wedge abcd-1=0$$

3.7 Cyclic-5

$$(\exists d)\,(\exists c)\,(\exists b)\,(\exists a)\;\; a+b+c+d+e=0 \wedge ab+ae+bc+cd+de= 0 \wedge abc+abe+ade+bcd+cde=0 \wedge abcd+abce+abde+acde+bcde=0 \wedge abcde-1=0$$

3.8 Ellipse A

$$(\exists y)\,(\exists x)\;\; x^2+y^2-1=0 \wedge b^2\left(x-c\right)^2+a^2y^2-a^2b^2=0 \wedge 0<a \wedge a<1 \wedge 0<b \wedge b< 1 \wedge 0 \le c \wedge c<1$$

Originally from [BH91]

3.9 Ellipse Problem

$$ab \neq 0 \wedge (\forall x)\,(\forall y)\;\; b^2\,(x-c)^2 + a^2\,(y-d)^2 - a^2b^2 = 0 \Rightarrow x^2 + y^2 \leq 1$$

Originally from [CMXY09]

3.10 Intersection

$$(\exists z)\;\; x^2 - 1/2\,y^2 - 1/2\,z^2 = 0 \wedge xz + yz - 2\,x = 0 \wedge z^2 - y = 0$$

Originally from [BH91]

3.11 Putnum Example

$$(\exists a)\,(\exists b)\,(\exists c)\,(\exists d)\;\; a^2+b^2-1 = 0 \wedge (c-10)^2+d^2-9 = 0 \wedge 2\,x-a-c = 0 \wedge 2+y-b-d = 0$$

Originally from [BG06]

3.12 Random A

$$(\exists z)\,(\forall y)\,(\exists x)\;\; 4\,x^2 + xy^2 - z + 1/4 = 0 \wedge 2\,x + y^2z + 1/2 = 0 \wedge x^2z - x/2 - y^2 = 0$$

Originally from [BH91]

3.13 Random B

$$(\exists x)\,(\forall y)\,(\exists z)\;\; 4\,x^2 + xy^2 - z + 1/4 = 0 \wedge 2\,x + y^2z + 1/2 = 0 \wedge x^2z - x/2 - y^2 = 0$$

Originally from [BH91]

3.14 SEIT Model

$$(\exists s)\,(\exists F)\,(\exists J)\,(\exists T)\;\; -Jbs - ds + d = 0 \wedge vF - (d+t)\,J = 0 \wedge bJ + cJT - (d+v+r)\,F + (1-q)\,tJ = 0 \wedge -JTb^2 + Jqt + Fr - Td = 0 \wedge 0 < F \wedge 0 < J \wedge 0 < T \wedge 0 < s \wedge 0 < b \wedge 0 < d \wedge 0 < v \wedge 0 < r \wedge 0 < t \wedge 0 < q \wedge 0 < b - c$$

Originally from [BG06]

3.15 Sharir 3-Cube

$$(\exists x_1)\,(\exists x_2)\,(\exists x_3)\;\; -25 \leq -2\,x_1 - 25\,x_2 + 10\,x_3 \wedge 2 \leq 25\,x_1 + 2\,x_2 + 10\,x_3 \wedge -25 \leq -2\,x_1 + 25\,x_2 + 10\,x_3 \wedge 2 \leq 25\,x_1 - 2\,x_2 + 10\,x_3 \wedge 0 \leq -x_2 - x_3 \wedge -2 \leq -x_2 + x_3$$

Originally from [Zie00]

3.16 Solotareff

$(\exists y)\,(\exists x)\;\; 3\,x^2 - a - 2\,x = 0 \wedge x^3 - ax - x^2 + a - 2\,b - 2 = 0 \wedge 3\,y^2 - a - 2\,y = 0 \wedge y^3 - ay - y^2 - a + 2 = 0 \wedge 0 \leq 4\,a - 1 \wedge 4\,a \leq 7 \wedge 0 \leq 4\,b + 3 \wedge 4\,b \leq 3 \wedge 0 \leq x + 1 \wedge x \leq 0 \wedge 0 \leq y \wedge y \leq 1$

Originally from [BH91]

3.17 Solotareff-3

$(\exists u)\,(\exists v)\;\; 0 < r \wedge 0 < r - 1 \wedge 0 < u + 1 \wedge u - v < 0 \wedge v < 1 \wedge 2\,ru + 3\,u^2 - a = 0 \wedge 2\,rv + 3\,v^2 - a = 0 \wedge ru^2 + u^3 - au + a - r - 1 = 0 \wedge rv^2 + v^3 - av - a - 2\,b + r + 1 = 0$

Originally from [CMXY09, Ach56]

3.18 Brown Davenport 1

$(\exists z_1)\,(\forall x_0)\,(\forall y_0)\;\; y_0 = y_1 \wedge x_0 = z_1 \vee y_0 = z_1 \wedge x_0 = x_1 \Rightarrow x_0 \leq 1/2 \wedge y_0 = 2\,x_0 \vee 1/2 < x_0 \wedge y_0 = 2 - 2\,x_0$

Originally from [BD07]

3.19 Brown Davenport 2

$(\exists z_2)\,(\forall x_1)\,(\forall y_1)\;\; y_1 = y_2 \wedge x_1 = z_2 \vee y_1 = z_2 \wedge x_1 = x_2 \Rightarrow (\exists z_1)\,(\forall x_0)\,(\forall y_0)\;\; y_0 = y_1 \wedge x_0 = z_1 \vee y_0 = z_1 \wedge x_0 = x_1 \Rightarrow x_0 \leq 1/2 \wedge y_0 = 2\,x_0 \vee 1/2 < x_0 \wedge y_0 = 2 - 2\,x_0$

Originally from [BD07]

3.20 Brown Davenport 3

$(\exists z_3)\,(\forall x_2)\,(\forall y_2)\;\; y_2 = y_3 \wedge x_2 = z_3 \vee y_2 = z_3 \wedge x_2 = x_3 \Rightarrow (\exists z_2)\,(\forall x_1)\,(\forall y_1)\;\; y_1 = y_2 \wedge x_1 = z_2 \vee y_1 = z_2 \wedge x_1 = x_2 \Rightarrow (\exists z_1)\,(\forall x_0)\,(\forall y_0)\;\; y_0 = y_1 \wedge x_0 = z_1 \vee y_0 = z_1 \wedge x_0 = x_1 \Rightarrow x_0 \leq 1/2 \wedge y_0 = 2\,x_0 \vee 1/2 < x_0 \wedge y_0 = 2 - 2\,x_0$

Originally from [BD07]

3.21 Brown Davenport 4

$(\exists z_4)\,(\forall x_3)\,(\forall y_3)\;\; y_3 = y_4 \wedge x_3 = z_4 \vee y_3 = z_4 \wedge x_3 = x_4 \Rightarrow (\exists z_3)\,(\forall x_2)\,(\forall y_2)\;\; y_2 = y_3 \wedge x_2 = z_3 \vee y_2 = z_3 \wedge x_2 = x_3 \Rightarrow (\exists z_2)\,(\forall x_1)\,(\forall y_1)\;\; y_1 = y_2 \wedge x_1 = z_2 \vee y_1 = z_2 \wedge x_1 = x_2 \Rightarrow (\exists z_1)\,(\forall x_0)\,(\forall y_0)\;\; y_0 = y_1 \wedge x_0 = z_1 \vee y_0 = z_1 \wedge x_0 = x_1 \Rightarrow x_0 \leq 1/2 \wedge y_0 = 2\,x_0 \vee 1/2 < x_0 \wedge y_0 = 2 - 2\,x_0$

Originally from [BD07]

3.22 Brown Davenport 5

$$(\exists z_5)\,(\forall x_4)\,(\forall y_4)\;\; y_4 = y_5 \wedge x_4 = z_5 \vee y_4 = z_5 \wedge x_4 = x_5 \Rightarrow (\exists z_4)\,(\forall x_3)\,(\forall y_3)\;\; y_3 = y_4 \wedge x_3 = z_4 \vee y_3 = z_4 \wedge x_3 = x_4 \Rightarrow (\exists z_3)\,(\forall x_2)\,(\forall y_2)\;\; y_2 = y_3 \wedge x_2 = z_3 \vee y_2 = z_3 \wedge x_2 = x_3 \Rightarrow (\exists z_2)\,(\forall x_1)\,(\forall y_1)\;\; y_1 = y_2 \wedge x_1 = z_2 \vee y_1 = z_2 \wedge x_1 = x_2 \Rightarrow (\exists z_1)\,(\forall x_0)\,(\forall y_0)\;\; y_0 = y_1 \wedge x_0 = z_1 \vee y_0 = z_1 \wedge x_0 = x_1 \Rightarrow x_0 \leq 1/2 \wedge y_0 = 2\,x_0 \vee 1/2 < x_0 \wedge y_0 = 2 - 2\,x_0$$

Originally from [BD07]

3.23 Collins and Johnson

$$(\exists r)\;\; 3\,a^2r - a^2 - 2\,ar + 2\,b^2 < 0 \wedge 0 < 3\,a^2r + 3\,b^2r - 2\,a^2 - 4\,ar - 2\,b^2 + 2\,a + r \wedge 0 \leq a - 1/2 \wedge 0 < b \wedge 0 < r \wedge r < 1$$

Originally from [HC91]

3.24 Collision Problem

$$(\exists t)\,(\exists x)\,(\exists y)\;\; 0 \leq \tfrac{17\,t}{16} - 6 \wedge \tfrac{17\,t}{16} \leq 10 \wedge 0 \leq x - \tfrac{17\,t}{16} + 1 \wedge x - \tfrac{17\,t}{16} \leq 1 \wedge 0 \leq y - \tfrac{17\,t}{16} + 9 \wedge y - \tfrac{17\,t}{16} \leq -7 \wedge 0 \leq (x-t)^2 + y^2 - 1$$

Originally from [HC91]

3.25 Davenport-Heintz

$$(\exists c)\,(\forall b)\,(\forall a)\;\; a - d = 0 \wedge b - c = 0 \vee a - c = 0 \wedge b - 1 = 0 \Rightarrow a^2 - b = 0$$

Originally from [DH88, HC91]

3.26 Edges Square Product

$$(\exists a)\,(\exists b)\,(\exists c)\;\; -ab + c + x = 0 \wedge -ac - x + y - 2 = 0 \wedge 0 \leq a \wedge -a \leq -2 \wedge 0 \leq b - 2 \wedge -b \leq -4 \wedge 0 \leq c + 1 \wedge -c \leq -1 \wedge 0 \leq x + 1 \wedge -x \leq -9 \wedge 0 \leq y + 6 \wedge -y \leq -6$$

Originally from [BG06]

3.27 NLRA Economics 1

$$(\forall v_1)\,(\forall v_2)\,(\forall v_3)\,(\forall v_4)\;\; v_1 < 0 \wedge 0 < v_2 \wedge v_2\,v_3 - 1 = v_4 \wedge v_4 = v_3\,v_1 \Rightarrow 0 < v_3 \wedge v_4 < 0$$

Originally from [MBD+18]

3.28 Off-Center Ellipse

$$a \neq 0 \wedge (\forall x)\,(\forall y)\;\; 16\,a^2y^2 - 8\,a^2y - 3\,a^2 + 4\,x^2 - 4\,x + 1 = 0 \Rightarrow x^2 + y^2 \leq 1$$

Originally from [AM88]

3.29 Parametric Parabola

$$(\exists x)\ \ ax^2 + bx + c = 0$$

Originally from [CMXY09]

3.30 Range of Lower Bounds

$$(\forall x)\,(\forall a)\,(\forall b)\,(\forall c)\,(\exists z)\ \ 0 < a \wedge az^2 + bz + c \neq 0 \Rightarrow 0 < ax^2 + bx + c - y$$

Originally from [DSS04]

3.31 Simplified Putnum

$$(\exists a)\,(\exists d)\ \ a^2 + d^2 - 4\,dy + 4\,y^2 - 1 = 0 \wedge a^2 - 4\,ax + d^2 + 4\,x^2 + 20\,a - 40\,x + 91 = 0$$

Originally from [BG06]

3.32 Simplified SEIT Model

$$(\exists J)\ \ d < 0 \wedge 0 < r \wedge 0 < t \wedge 0 < q \wedge 0 < b - c \wedge 0 <$$
$$v \wedge 0 < J \wedge 0 < b \wedge 0 < c \wedge bJ + d \neq 0 \wedge -v \neq 0 \wedge 0 < (d+t)\,Jv \wedge vc \neq 0 \wedge 0 < d\,(bJ+d) \wedge 0 <$$
$$(bJ+d)\,cv\,\left(Jbqtv + Jbd^2 + Jbdr + Jbdt + Jbdv + Jbtv + dqtv - bdv + d^3 + d^2r + d^2t + d^2v + drt\right) \wedge$$
$$0 =$$
$$-\,(bJ+d)\,cv^3d\,\left(J^2bcd + J^2bct + J^2bcv + Jbqtv - Jbcv + Jbd^2 + Jbdr + Jbdt + Jbdv + Jbrt + Jcd^2 + Jcdt$$

Originally from [BG06]

3.33 Simplified YangXia

$$(\exists b)\ \ -b/2 \neq 0 \wedge 0 < R \wedge 0 < b \wedge 0 < h \wedge 1/16\,a^2h^2b^4 - 1/32\,a^2b^6 - 1/8\,a^2R^2h^2b^2 -$$
$$1/8\,R^2h^2b^4 + \frac{b^8}{64} + \frac{a^4b^4}{64} + 1/4\,R^4h^4 = 0 \wedge 0 < -1/4\,\left(2\,Rh - ab - b^2\right)b \wedge 0 <$$
$$1/2\,Rhb \wedge 0 < 1/4\,\left(2\,Rh + ab - b^2\right)b \wedge 0 < 1/4\,\left(2\,Rh - ab + b^2\right)b$$

Originally from [BG06]

3.34 Whitney Umbrella

$$(\exists u)\,(\exists v)\ \ -uv + x = 0 \wedge y - v = 0 \wedge -u^2 + z = 0$$

Originally from [CMXY09]

3.35 X-axis Ellipse Problem

$$ab \neq 0 \wedge (\forall x)\,(\forall y)\ \ b^2\,(x-c)^2 + a^2y^2 - a^2b^2 = 0 \Rightarrow x^2 + y^2 \leq 1$$

Originally from [DSS04]

3.36 A Real Implicitization Problem

$$(\exists u)\,(\exists v)\quad -uv+x=0 \wedge -uv^2+y=0 \wedge -u^2+z=0$$

Originally from [DSS04]

3.37 Applied Mechanics Problems 1a

$$(\forall F_1)\,(\forall F_2)\,(\forall F_3)\quad \tfrac{285}{2} \le F_1 \wedge F_1 \le \tfrac{315}{2} \wedge 665 \le F_2 \wedge F_2 \le 735 \wedge 285 \le F_3 \wedge F_3 \le 315 \Rightarrow N_1 \le F_1 - F_2 + F_3 \wedge F_1 - F_2 + F_3 \le N_2$$

Originally from [Ioa19]

3.38 Applied Mechanics Problems 1b

$$(\forall F_1)\,(\forall F_2)\,(\forall F_3)\quad \tfrac{285}{2} \le F_1 \wedge F_1 \le \tfrac{315}{2} \wedge 665 \le F_2 \wedge F_2 \le 735 \wedge \tfrac{1045}{2} \le F_3 \wedge F_3 \le \tfrac{1155}{2} \Rightarrow N_1 \le F_1 - F_2 + F_3 \wedge F_1 - F_2 + F_3 \le N_2$$

Originally from [Ioa19]

3.39 Applied Mechanics Problems 1c

$$(\forall F_1)\,(\forall F_2)\,(\forall F_3)\quad \tfrac{285}{2} \le F_1 \wedge F_1 \le \tfrac{315}{2} \wedge 665 \le F_2 \wedge F_2 \le 735 \wedge \tfrac{1045}{2} \le F_3 \wedge F_3 \le \tfrac{1155}{2} \Rightarrow N_1 \le F_1 - F_2 + F_3 \wedge F_1 - F_2 + F_3 \le N_2$$

Originally from [Ioa19]

3.40 Applied Mechanics Problems 2a

$$(\forall p)\quad 150 - 150\,p \le F_1 \wedge F_1 \le 150 + 150\,p \wedge 700 - 700\,p \le F_2 \wedge F_2 \le 700 + 700\,p \wedge 300 - 300\,p \le F_3 \wedge F_3 \le 300 + 300\,p \Rightarrow N_1 \le F_1 - F_2 + F_3 \wedge F_1 - F_2 + F_3 \le N_2$$

Originally from [Ioa19]

3.41 Applied Mechanics Problems 2b

$$(\forall p)\quad 150 - 150\,p \le F_1 \wedge F_1 \le 150 + 150\,p \wedge 700 - 700\,p \le F_2 \wedge F_2 \le 700 + 700\,p \wedge 550 - 550\,p \le F_3 \wedge F_3 \le 550 + 550\,p \Rightarrow N_1 \le F_1 - F_2 + F_3 \wedge F_1 - F_2 + F_3 \le N_2$$

Originally from [Ioa19]

3.42 Applied Mechanics Problems 2c

$$(\forall p)\quad 150 - 150\,p \le F_1 \wedge F_1 \le 150 + 150\,p \wedge 700 - 700\,p \le F_2 \wedge F_2 \le 700 + 700\,p \wedge 1200 - 1200\,p \le F_3 \wedge F_3 \le 1200 + 1200\,p \Rightarrow N_1 \le F_1 - F_2 + F_3 \wedge F_1 - F_2 + F_3 \le N_2$$

Originally from [Ioa19]

3.43 Applied Mechanics Problems 3

$$(\forall Fp)\ \ 0 < \mathit{N1} \wedge 0 < \mathit{N2} \wedge -1 \leq Fp \wedge Fp \leq 1 \Rightarrow \mathit{N1}^2 \leq \left(\frac{47721\,Fp}{32029} + \frac{361085}{1914}\right)^2 + \left(-\frac{48056\,Fp}{28293} - \frac{332794}{2615}\right)^2 \wedge \left(\frac{47721\,Fp}{32029} + \frac{361085}{1914}\right)^2 + \left(-\frac{48056\,Fp}{28293} - \frac{332794}{2615}\right)^2 \leq \mathit{N2}^2$$

Originally from [Ioa19]

3.44 Applied Mechanics Problems 3x

$$(\forall F_1)\,(\forall F_2)\,(\forall F_3)\,(\forall F_4)\ \ 129 \leq F_1 \wedge F_1 \leq 131 \wedge 59 \leq F_2 \wedge F_2 \leq 61 \wedge 109 \leq F_3 \wedge F_3 \leq 111 \wedge 99 \leq F_4 \wedge F_4 \leq 101 \Rightarrow \mathit{Nx1} \leq \frac{34641\,F_1}{40000} - \frac{17101\,F_2}{50000} + \frac{19475\,F_4}{20162} \wedge \frac{34641\,F_1}{40000} - \frac{17101\,F_2}{50000} + \frac{19475\,F_4}{20162} \leq \mathit{Nx2}$$

Originally from [Ioa19]

3.45 Applied Mechanics Problems 3y

$$(\forall F_1)\,(\forall F_2)\,(\forall F_3)\,(\forall F_4)\ \ 129 \leq F_1 \wedge F_1 \leq 131 \wedge 59 \leq F_2 \wedge F_2 \leq 61 \wedge 109 \leq F_3 \wedge F_3 \leq 111 \wedge 99 \leq F_4 \wedge F_4 \leq 101 \Rightarrow \mathit{Ny1} \leq F_1/2 - \frac{58463\,F_2}{62215} - F_3 - \frac{5378\,F_4}{20779} \wedge F_1/2 - \frac{58463\,F_2}{62215} - F_3 - \frac{5378\,F_4}{20779} \leq \mathit{Ny2}$$

Originally from [Ioa19]

3.46 Applied Mechanics Problems 4a

$$(\exists q)\ \ -1 \leq q \wedge q \leq 1 \Rightarrow 525000\,\left(2 + \sqrt{2}\right) u_1 - 525000\,\sqrt{2} u_2 - 525000\,\sqrt{2} v_2 = 0 \wedge -525000\,\sqrt{2} u_1 + 52500\,\sqrt{2}\,(q + 20)\,u_2 - 52500\,\sqrt{2} v_2 = 0 \wedge -525000\,\sqrt{2} u_1 - 52500\,\sqrt{2} u_2 + 52500\,\sqrt{2}\,(q + 20)\,v_2 = -10$$

Originally from [Ioa19]

3.47 Applied Mechanics Problems 4b

$$(\exists q)\ \ -1 \leq q \wedge q \leq 1 \Rightarrow \left(\sqrt{2} q + 20 + 10\,\sqrt{2}\right) u_1 - \sqrt{2}\,(q + 10)\,(u_2 + v_2) = 0 \wedge (q + 10)\,(u_1 - 2\,u_2) = 0 \wedge 5250\,\sqrt{2}\,(q + 10)\,(u_1 - 2\,v_2) = 1$$

Originally from [Ioa19]

3.48 Ball and Circular Cylinder

$$(\exists z)\,(\exists x)\,(\exists y)\ \ x^2 + y^2 + z^2 < 1 \wedge x^2 + (y + z - 2)^2 < 1$$

Originally from [McC88]

3.49 Joukowsky Original Formulation

$$(\forall a)\,(\forall b)\,(\forall c)\,(\forall d)\;\; a\left(c^2+d^2\right)\left(a^2+b^2+1\right)-c\left(a^2+b^2\right)\left(c^2+d^2+1\right)= 0 \wedge b\left(c^2+d^2\right)\left(a^2+b^2-1\right)-d\left(a^2+b^2\right)\left(c^2+d^2-1\right)=0 \wedge 0<bd \wedge 0<c^2+d^2-1 \Rightarrow a-c=0 \wedge b-d=0$$

3.50 Joukowsky Separate Clauses

$$(\exists a)\,(\exists b)\,(\exists c)\,(\exists d)\;\; a\left(c^2+d^2\right)\left(a^2+b^2+1\right)-c\left(a^2+b^2\right)\left(c^2+d^2+1\right)=0 \wedge b\left(c^2+d^2\right)\left(a^2+b^2-1\right)-d\left(a^2+b^2\right)\left(c^2+d^2-1\right)=0 \wedge 0<bd \wedge 0<c^2+d^2-1 \wedge a-c\neq 0$$

3.51 Joukowsky Upper Half Plane

$$(\forall a)\,(\forall b)\,(\forall c)\,(\forall d)\;\; a\left(c^2+d^2\right)\left(a^2+b^2+1\right)-c\left(a^2+b^2\right)\left(c^2+d^2+1\right)= 0 \wedge b\left(c^2+d^2\right)\left(a^2+b^2-1\right)-d\left(a^2+b^2\right)\left(c^2+d^2-1\right)=0 \wedge 0<b \wedge 0<d \Rightarrow a-c= 0 \wedge b-d=0$$

3.52 Piano Movers Problem (Wang)

$$(\exists a)\,(\exists b)\,(\exists c)\,(\exists d)\;\; a^2+b^2=r^2 \wedge 0\leq a \wedge b<0 \wedge 1\leq c \wedge d<-1 \wedge c-(1+b)\,(c-a)= 0 \wedge d-(1-a)\,(d-b)=0$$

Originally from [WDEB13]

3.53 Positive Invariant Sets 1

$$(\exists q)\,(\forall x_1)\,(\forall x_2)\,(\forall x_3)\;\; 0<q \wedge 0<l \wedge 0<a \wedge 0<b \wedge 0< c \wedge 1/2\,bc\left(-2\,{x_1}^2x_2+2\,{x_1}^2-2\,x_1\,x_3+2\,\left(x_2-2\right)\left(-ax_2+{x_1}^2\right)\right)+bx_3\,\left(cx_1-x_3\right)\leq -q\left(1/2\,bc\left({x_1}^2+\left(x_2-2\right)^2\right)+1/2\,{x_3}^2-l\right)$$

Originally from [RVR19]

3.54 Positive Invariant Sets 2

$$(\exists q)\,(\forall x_1)\,(\forall x_2)\,(\forall x_3)\;\; 0<l \wedge 0<q \wedge (4\,bkr_1\,x_4-2\,bk{x_4}^2+4\,kq{r_1}^2-4\,kqr_1\,x_4+kq{x_2}^2+kq{x_3}^2+kq{x_4}^2-2\,kr_1\,{x_1}^2+4\,kr_1\,x_1\,x_2+2\,kr_2\,x_1\,x_3+ \ldots$$

$$0 \vee 4\,bkr_1\,x_4-2\,bk{x_4}^2+4\,kq{r_1}^2-4\,kqr_1\,x_4+kq{x_2}^2+kq{x_3}^2+kq{x_4}^2-2\,kr_1\,{x_1}^2+ 4\,kr_1\,x_1\,x_2+2\,kr_2\,x_1\,x_3+qr_1\,{x_1}^2-klq-4\,kx_1\,x_2-2\,k{x_3}^2=0 \wedge k\neq 0$$

Originally from [RVR19]

3.55 Positive Invariant Sets 3

$$(\exists q)\,(\exists l)\,(\exists p_1)\,(\exists x_{40})\,(\forall x_1)\,(\forall x_2)\,(\forall x_3)\;\; 0<l \wedge 0<p_1 \wedge 0< q \wedge -2\,p_1\,{x_1}^2k+2\,p_1\,x_1\,x_2\,k+2\,r_1\,x_1\,x_2-2\,{x_2}^2-2\,x_1\,x_2\,x_4+2\,r_2\,x_1\,x_3-2\,{x_3}^2+ 2\,\left(x_4-x_{40}\right)\left(-bx_4+x_1\,x_2\right)\leq -q\left(p_1\,{x_1}^2+{x_2}^2+{x_3}^2+\left(x_4-x_{40}\right)^2-l\right)$$

Originally from [RVR19]

3.56 Positive Invariant Sets 4

$$(\exists q)\,(\exists l)\,(\exists x_{40})\,(\forall x_1)\,(\forall x_2)\,(\forall x_3)\ \ 0 < l \wedge 0 < p_1 \wedge 0 < q \wedge -2\,p_1\,{x_1}^2 k + 2\,p_1\,x_1\,x_2\,k + 2\,r_1\,x_1\,x_2 - 2\,{x_2}^2 - 2\,x_1\,x_2\,x_4 + 2\,r_2\,x_1\,x_3 - 2\,{x_3}^2 + 2\,(x_4 - x_{40})\,(-bx_4 + x_1\,x_2) \leq -q\left(p_1\,{x_1}^2 + {x_2}^2 + {x_3}^2 + (x_4 - x_{40})^2 - l\right)$$

Originally from [RVR19]

3.57 Positive Invariant Sets 5

$$(\exists q)\,(\exists l)\,(\exists p_1)\,(\forall x_1)\,(\forall x_2)\,(\forall x_3)\ \ 0 < l \wedge 0 < p_1 \wedge 0 < q \wedge -2\,p_1\,{x_1}^2 k + 2\,p_1\,x_1\,x_2\,k + 2\,r_1\,x_1\,x_2 - 2\,{x_2}^2 - 2\,x_1\,x_2\,x_4 + 2\,r_2\,x_1\,x_3 - 2\,{x_3}^2 + 2\,(x_4 - x_{40})\,(-bx_4 + x_1\,x_2) \leq -q\left(p_1\,{x_1}^2 + {x_2}^2 + {x_3}^2 + (x_4 - x_{40})^2 - l\right)$$

Originally from [RVR19]

3.58 Positive Invariant Sets 6

$$(\exists q)\,(\exists l)\,(\forall x_1)\,(\forall x_2)\,(\forall x_3)\ \ 0 < l \wedge 0 < p_1 \wedge 0 < q \wedge -2\,p_1\,{x_1}^2 k + 2\,p_1\,x_1\,x_2\,k + 2\,r_1\,x_1\,x_2 - 2\,{x_2}^2 - 2\,x_1\,x_2\,x_4 + 2\,r_2\,x_1\,x_3 - 2\,{x_3}^2 + 2\,(x_4 - x_{40})\,(-bx_4 + x_1\,x_2) \leq -q\left(p_1\,{x_1}^2 + {x_2}^2 + {x_3}^2 + (x_4 - x_{40})^2 - l\right)$$

Originally from [RVR19]

3.59 Positive Invariant Sets 7

$$(\exists q)\,(\forall x_1)\,(\forall x_2)\,(\forall x_3)\ \ 0 < l \wedge 0 < p_1 \wedge 0 < q \wedge -2\,p_1\,{x_1}^2 k + 2\,p_1\,x_1\,x_2\,k + 2\,r_1\,x_1\,x_2 - 2\,{x_2}^2 - 2\,x_1\,x_2\,x_4 + 2\,r_2\,x_1\,x_3 - 2\,{x_3}^2 + 2\,(x_4 - x_{40})\,(-bx_4 + x_1\,x_2) \leq -q\left(p_1\,{x_1}^2 + {x_2}^2 + {x_3}^2 + (x_4 - x_{40})^2 - l\right)$$

Originally from [RVR19]

3.60 Quantified Linear System

$$(\forall x)\,(\forall z)\,(\exists y)\,(\exists w)\ \ x + y \leq z \wedge w \leq y \wedge z \leq zx - y$$

Originally from [RESW14]

3.61 Simplified Edges Square Product

$$(\exists a)\ \ -a \leq 0 \wedge a \leq 2 \wedge 0 \leq x + 1 \wedge x \leq 9 \wedge 0 \leq y + 6 \wedge y \leq 6 \wedge 0 \leq -\left(a^2 + 1\right)\left(2\,a^2 - ax - y + 2\right) \wedge (ay - x)\left(a^2 + 1\right) \leq \left(a^2 + 1\right)^2 \wedge a^2 + 1 \neq 0 \wedge 0 \leq \left(a^2 + 1\right)\left(a^2 + ay - x + 1\right) \wedge \left(a^2 + 1\right)(ax + y) \leq 4\,\left(a^2 + 1\right)^2$$

Originally from [BG06]

3.62 2D Euclidean Space Axiomatisation 1

$$(\exists x_1)\,(\exists y_1)\,(\exists x_2)\,(\exists y_2)\;\; (x_1 - x_2)^2 + (y_1 - y_2)^2 \leq 1$$

Originally from [DA19]

3.63 2D Euclidean Space Axiomatisation 2

$$(\exists x_1)\,(\exists y_1)\,(\exists x_2)\,(\exists y_2)\,(\exists x_3)\,(\exists y_3)\;\; (x_1 - x_2)^2 + (y_1 - y_2)^2 \leq 1 \wedge (x_2 - x_3)^2 + (y_2 - y_3)^2 \leq 1 \wedge 4 < (x_1 - x_3)^2 + (y_1 - y_3)^2$$

Originally from [DA19]

3.64 2D Euclidean Space Axiomatisation 3

$$(\exists x_1)\,(\exists y_1)\,(\exists x_2)\,(\exists y_2)\,(\exists x_3)\,(\exists y_3)\,(\exists x_4)\,(\exists y_4)\;\; (x_1 - x_2)^2 + (y_1 - y_2)^2 \leq 1 \wedge (x_2 - x_3)^2 + (y_2 - y_3)^2 \leq 4 \wedge (x_3 - x_4)^2 + (y_3 - y_4)^2 \leq 1 \wedge 16 < (x_1 - x_4)^2 + (y_1 - y_4)^2$$

Originally from [DA19]

3.65 Piano Movers Problem (Davenport)

$$0 < x \vee 0 < w \vee y < 0 \vee z < 0 \vee (\exists t)\;\; 0 < t \wedge t < 1 \wedge x + t\,(w - x) < -1 \wedge 1 < y + t\,(z - y)$$

Originally from [WDEB13]

3.66 Piano Movers Problem (Yang, Zheng)

$$(\forall x)\;\; 0 < 4\,x^8 - 4\,(L-3)\,x^6 - 2\,(3\,L - 6)\,x^4 - 2\,(L-3)\,x^2 + 1$$

Originally from [WDEB13]

3.67 Termination of Term Rewrite System

$$(\exists r)\,(\forall x)\,(\forall y)\;\; 0 < x - r \wedge 0 < y - r \Rightarrow 0 < x^2\,(1 + 2\,y)^2 - y^2\,(2\,x^2 + 1)$$

Originally from [HC91]

3.68 RA Triangle Hyp Longer Than Sum of Sides

$$(\forall a)\,(\forall b)\;\; 0 < a \wedge 0 < b \wedge a^2 + b^2 = c^2 \Rightarrow a + b < c$$

4 Economics QE Examples

The paper [MBD+18] discusses the genesis of various QE problems from economics. While various QE questions can be posed from any one set of assumptions and hypotheses (and indeed a select couple can be found in the examples above), the following are the constituent assumption/hypothesis pairs A and H themselves, from which one may pose eg. the "theorem" $\forall \mathbf{x}\; A(\mathbf{x}) \Rightarrow H(\mathbf{x})$.

4.1 0058

Assumptions:

$$true$$

Hypotheses:

$v_2 \le v_1 \wedge v_3 \le v_1 \wedge v_4 \le v_1 \wedge v_5 \le v_1 \wedge v_6 \le v_1 \wedge v_7 \le v_1 \wedge v_8 \le v_1 \wedge v_9 \le v_1 \wedge v_{10} \le v_1 \wedge v_{11} \le$
$v_1 \wedge v_{12} \le v_1 \wedge v_{13} \le v_1 \wedge v_{14} \le v_1 \wedge v_{15} \le v_1 \wedge v_{16} \le v_1 \wedge v_{17} \le v_1 \wedge v_{18} \le v_1 \wedge v_{19} \le v_1 \wedge v_{20} \le$
$v_1 \wedge v_{21} \le v_1 \wedge 1 \le 2\,v_1 \wedge v_1 \le v_2 \wedge v_3 \le v_2 \wedge v_4 \le v_2 \wedge v_5 \le v_2 \wedge v_6 \le v_2 \wedge v_7 \le v_2 \wedge v_8 \le$
$v_2 \wedge v_9 \le v_2 \wedge v_{10} \le v_2 \wedge v_{11} \le v_2 \wedge v_{12} \le v_2 \wedge v_{13} \le v_2 \wedge v_{14} \le v_2 \wedge v_{15} \le v_2 \wedge v_{16} \le v_2 \wedge v_{17} \le$
$v_2 \wedge v_{18} \le v_2 \wedge v_{19} \le v_2 \wedge v_{20} \le v_2 \wedge v_{21} \le v_2 \wedge 1 \le 2\,v_2 \wedge 2 \le 2\,v_1 + v_2 \wedge 2 \le v_1 + 2\,v_2 \wedge v_1 \le$
$v_3 \wedge v_2 \le v_3 \wedge v_4 \le v_3 \wedge v_5 \le v_3 \wedge v_6 \le v_3 \wedge v_7 \le v_3 \wedge v_8 \le v_3 \wedge v_9 \le v_3 \wedge v_{10} \le v_3 \wedge v_{11} \le$
$v_3 \wedge v_{12} \le v_3 \wedge v_{13} \le v_3 \wedge v_{14} \le v_3 \wedge v_{15} \le v_3 \wedge v_{16} \le v_3 \wedge v_{17} \le v_3 \wedge v_{18} \le v_3 \wedge v_{19} \le v_3 \wedge v_{20} \le$
$v_3 \wedge v_{21} \le v_3 \wedge 1 \le 2\,v_3 \wedge 2 \le 2\,v_1 + v_3 \wedge 2 \le 2\,v_2 + v_3 \wedge 2 \le v_1 + 2\,v_3 \wedge 2 \le v_2 + 2\,v_3 \wedge v_1 \le$
$v_4 \wedge v_2 \le v_4 \wedge v_3 \le v_4 \wedge v_5 \le v_4 \wedge v_6 \le v_4 \wedge v_7 \le v_4 \wedge v_8 \le v_4 \wedge v_9 \le v_4 \wedge v_{10} \le v_4 \wedge v_{11} \le$
$v_4 \wedge v_{12} \le v_4 \wedge v_{13} \le v_4 \wedge v_{14} \le v_4 \wedge v_{15} \le v_4 \wedge v_{16} \le v_4 \wedge v_{17} \le v_4 \wedge v_{18} \le v_4 \wedge v_{19} \le v_4 \wedge v_{20} \le$
$v_4 \wedge v_{21} \le v_4 \wedge 1 \le 2\,v_4 \wedge 2 \le 2\,v_1 + v_4 \wedge 2 \le 2\,v_2 + v_4 \wedge 2 \le 2\,v_3 + v_4 \wedge 2 \le v_1 + 2\,v_4 \wedge 2 \le$
$v_2 + 2\,v_4 \wedge 2 \le v_3 + 2\,v_4 \wedge v_1 \le v_5 \wedge v_2 \le v_5 \wedge v_3 \le v_5 \wedge v_4 \le v_5 \wedge v_6 \le v_5 \wedge v_7 \le v_5 \wedge v_8 \le$
$v_5 \wedge v_9 \le v_5 \wedge v_{10} \le v_5 \wedge v_{11} \le v_5 \wedge v_{12} \le v_5 \wedge v_{13} \le v_5 \wedge v_{14} \le v_5 \wedge v_{15} \le v_5 \wedge v_{16} \le v_5 \wedge v_{17} \le$
$v_5 \wedge v_{18} \le v_5 \wedge v_{19} \le v_5 \wedge v_{20} \le v_5 \wedge v_{21} \le v_5 \wedge 1 \le 2\,v_5 \wedge 2 \le 2\,v_1 + v_5 \wedge 2 \le 2\,v_2 + v_5 \wedge 2 \le$
$2\,v_3 + v_5 \wedge 2 \le 2\,v_4 + v_5 \wedge 2 \le v_1 + 2\,v_5 \wedge 2 \le v_2 + 2\,v_5 \wedge 2 \le v_3 + 2\,v_5 \wedge 2 \le v_4 + 2\,v_5 \wedge v_1 \le$
$v_6 \wedge v_2 \le v_6 \wedge v_3 \le v_6 \wedge v_4 \le v_6 \wedge v_5 \le v_6 \wedge v_7 \le v_6 \wedge v_8 \le v_6 \wedge v_9 \le v_6 \wedge v_{10} \le v_6 \wedge v_{11} \le$
$v_6 \wedge v_{12} \le v_6 \wedge v_{13} \le v_6 \wedge v_{14} \le v_6 \wedge v_{15} \le v_6 \wedge v_{16} \le v_6 \wedge v_{17} \le v_6 \wedge v_{18} \le v_6 \wedge v_{19} \le v_6 \wedge v_{20} \le$
$v_6 \wedge v_{21} \le v_6 \wedge 1 \le 2\,v_6 \wedge 2 \le 2\,v_1 + v_6 \wedge 2 \le 2\,v_2 + v_6 \wedge 2 \le 2\,v_3 + v_6 \wedge 2 \le 2\,v_4 + v_6 \wedge 2 \le$
$2\,v_5 + v_6 \wedge 2 \le v_1 + 2\,v_6 \wedge 2 \le v_2 + 2\,v_6 \wedge 2 \le v_3 + 2\,v_6 \wedge 2 \le v_4 + 2\,v_6 \wedge 2 \le v_5 + 2\,v_6 \wedge v_1 \le$
$v_7 \wedge v_2 \le v_7 \wedge v_3 \le v_7 \wedge v_4 \le v_7 \wedge v_5 \le v_7 \wedge v_6 \le v_7 \wedge v_8 \le v_7 \wedge v_9 \le v_7 \wedge v_{10} \le v_7 \wedge v_{11} \le$
$v_7 \wedge v_{12} \le v_7 \wedge v_{13} \le v_7 \wedge v_{14} \le v_7 \wedge v_{15} \le v_7 \wedge v_{16} \le v_7 \wedge v_{17} \le v_7 \wedge v_{18} \le v_7 \wedge v_{19} \le v_7 \wedge v_{20} \le$
$v_7 \wedge v_{21} \le v_7 \wedge 1 \le 2\,v_7 \wedge 2 \le 2\,v_1 + v_7 \wedge 2 \le 2\,v_2 + v_7 \wedge 2 \le 2\,v_3 + v_7 \wedge 2 \le 2\,v_4 + v_7 \wedge 2 \le$
$2\,v_5 + v_7 \wedge 2 \le 2\,v_6 + v_7 \wedge 2 \le v_1 + 2\,v_7 \wedge 2 \le v_2 + 2\,v_7 \wedge 2 \le v_3 + 2\,v_7 \wedge 2 \le v_4 + 2\,v_7 \wedge 2 \le$
$v_5 + 2\,v_7 \wedge 2 \le v_6 + 2\,v_7 \wedge v_1 \le v_8 \wedge v_2 \le v_8 \wedge v_3 \le v_8 \wedge v_4 \le v_8 \wedge v_5 \le v_8 \wedge v_6 \le v_8 \wedge v_7 \le$
$v_8 \wedge v_9 \le v_8 \wedge v_{10} \le v_8 \wedge v_{11} \le v_8 \wedge v_{12} \le v_8 \wedge v_{13} \le v_8 \wedge v_{14} \le v_8 \wedge v_{15} \le v_8 \wedge v_{16} \le v_8 \wedge v_{17} \le$
$v_8 \wedge v_{18} \le v_8 \wedge v_{19} \le v_8 \wedge v_{20} \le v_8 \wedge v_{21} \le v_8 \wedge 1 \le 2\,v_8 \wedge 2 \le 2\,v_1 + v_8 \wedge 2 \le 2\,v_2 + v_8 \wedge 2 \le$
$2\,v_3 + v_8 \wedge 2 \le 2\,v_4 + v_8 \wedge 2 \le 2\,v_5 + v_8 \wedge 2 \le 2\,v_6 + v_8 \wedge 2 \le 2\,v_7 + v_8 \wedge 2 \le v_1 + 2\,v_8 \wedge 2 \le$
$v_2 + 2\,v_8 \wedge 2 \le v_3 + 2\,v_8 \wedge 2 \le v_4 + 2\,v_8 \wedge 2 \le v_5 + 2\,v_8 \wedge 2 \le v_6 + 2\,v_8 \wedge 2 \le v_7 + 2\,v_8 \wedge v_1 \le$
$v_9 \wedge v_2 \le v_9 \wedge v_3 \le v_9 \wedge v_4 \le v_9 \wedge v_5 \le v_9 \wedge v_6 \le v_9 \wedge v_7 \le v_9 \wedge v_8 \le v_9 \wedge v_{10} \le v_9 \wedge v_{11} \le$
$v_9 \wedge v_{12} \le v_9 \wedge v_{13} \le v_9 \wedge v_{14} \le v_9 \wedge v_{15} \le v_9 \wedge v_{16} \le v_9 \wedge v_{17} \le v_9 \wedge v_{18} \le v_9 \wedge v_{19} \le v_9 \wedge v_{20} \le$
$v_9 \wedge v_{21} \le v_9 \wedge 1 \le 2\,v_9 \wedge 2 \le 2\,v_1 + v_9 \wedge 2 \le 2\,v_2 + v_9 \wedge 2 \le 2\,v_3 + v_9 \wedge 2 \le 2\,v_4 + v_9 \wedge 2 \le$
$2\,v_5 + v_9 \wedge 2 \le 2\,v_6 + v_9 \wedge 2 \le 2\,v_7 + v_9 \wedge 2 \le 2\,v_8 + v_9 \wedge 2 \le v_1 + 2\,v_9 \wedge 2 \le v_2 + 2\,v_9 \wedge 2 \le$
$v_3 + 2\,v_9 \wedge 2 \le v_4 + 2\,v_9 \wedge 2 \le v_5 + 2\,v_9 \wedge 2 \le v_6 + 2\,v_9 \wedge 2 \le v_7 + 2\,v_9 \wedge 2 \le v_8 + 2\,v_9 \wedge v_1 \le$
$v_{10} \wedge v_2 \le v_{10} \wedge v_3 \le v_{10} \wedge v_4 \le v_{10} \wedge v_5 \le v_{10} \wedge v_6 \le v_{10} \wedge v_7 \le v_{10} \wedge v_8 \le v_{10} \wedge v_9 \le v_{10} \wedge v_{11} \le$
$v_{10} \wedge v_{12} \le v_{10} \wedge v_{13} \le v_{10} \wedge v_{14} \le v_{10} \wedge v_{15} \le v_{10} \wedge v_{16} \le v_{10} \wedge v_{17} \le v_{10} \wedge v_{18} \le v_{10} \wedge v_{19} \le$
$v_{10} \wedge v_{20} \le v_{10} \wedge v_{21} \le v_{10} \wedge 1 \le 2\,v_{10} \wedge 2 \le 2\,v_1 + v_{10} \wedge 2 \le v_{10} + 2\,v_2 \wedge 2 \le v_{10} + 2\,v_3 \wedge 2 \le$
$v_{10} + 2\,v_4 \wedge 2 \le v_{10} + 2\,v_5 \wedge 2 \le v_{10} + 2\,v_6 \wedge 2 \le v_{10} + 2\,v_7 \wedge 2 \le v_{10} + 2\,v_8 \wedge 2 \le v_{10} + 2\,v_9 \wedge 2 \le$

$v_1 + 2\,v_{10} \wedge 2 \leq 2\,v_{10} + v_2 \wedge 2 \leq 2\,v_{10} + v_3 \wedge 2 \leq 2\,v_{10} + v_4 \wedge 2 \leq 2\,v_{10} + v_5 \wedge 2 \leq 2\,v_{10} + v_6 \wedge 2 \leq$
$2\,v_{10} + v_7 \wedge 2 \leq 2\,v_{10} + v_8 \wedge 2 \leq 2\,v_{10} + v_9 \wedge v_1 \leq v_{11} \wedge v_2 \leq v_{11} \wedge v_3 \leq v_{11} \wedge v_4 \leq v_{11} \wedge v_5 \leq$
$v_{11} \wedge v_6 \leq v_{11} \wedge v_7 \leq v_{11} \wedge v_8 \leq v_{11} \wedge v_9 \leq v_{11} \wedge v_{10} \leq v_{11} \wedge v_{12} \leq v_{11} \wedge v_{13} \leq v_{11} \wedge v_{14} \leq$
$v_{11} \wedge v_{15} \leq v_{11} \wedge v_{16} \leq v_{11} \wedge v_{17} \leq v_{11} \wedge v_{18} \leq v_{11} \wedge v_{19} \leq v_{11} \wedge v_{20} \leq v_{11} \wedge v_{21} \leq v_{11} \wedge 1 \leq$
$2\,v_{11} \wedge 2 \leq 2\,v_1 + v_{11} \wedge 2 \leq v_{11} + 2\,v_2 \wedge 2 \leq v_{11} + 2\,v_3 \wedge 2 \leq v_{11} + 2\,v_4 \wedge 2 \leq v_{11} + 2\,v_5 \wedge 2 \leq$
$v_{11} + 2\,v_6 \wedge 2 \leq v_{11} + 2\,v_7 \wedge 2 \leq v_{11} + 2\,v_8 \wedge 2 \leq v_{11} + 2\,v_9 \wedge 2 \leq 2\,v_{10} + v_{11} \wedge 2 \leq v_1 + 2\,v_{11} \wedge 2 \leq$
$2\,v_{11} + v_2 \wedge 2 \leq 2\,v_{11} + v_3 \wedge 2 \leq 2\,v_{11} + v_4 \wedge 2 \leq 2\,v_{11} + v_5 \wedge 2 \leq 2\,v_{11} + v_6 \wedge 2 \leq 2\,v_{11} + v_7 \wedge 2 \leq$
$2\,v_{11} + v_8 \wedge 2 \leq 2\,v_{11} + v_9 \wedge 2 \leq v_{10} + 2\,v_{11} \wedge v_1 \leq v_{12} \wedge v_2 \leq v_{12} \wedge v_3 \leq v_{12} \wedge v_4 \leq v_{12} \wedge v_5 \leq$
$v_{12} \wedge v_6 \leq v_{12} \wedge v_7 \leq v_{12} \wedge v_8 \leq v_{12} \wedge v_9 \leq v_{12} \wedge v_{10} \leq v_{12} \wedge v_{11} \leq v_{12} \wedge v_{13} \leq v_{12} \wedge v_{14} \leq$
$v_{12} \wedge v_{15} \leq v_{12} \wedge v_{16} \leq v_{12} \wedge v_{17} \leq v_{12} \wedge v_{18} \leq v_{12} \wedge v_{19} \leq v_{12} \wedge v_{20} \leq v_{12} \wedge v_{21} \leq v_{12} \wedge 1 \leq$
$2\,v_{12} \wedge 2 \leq 2\,v_1 + v_{12} \wedge 2 \leq v_{12} + 2\,v_2 \wedge 2 \leq v_{12} + 2\,v_3 \wedge 2 \leq v_{12} + 2\,v_4 \wedge 2 \leq v_{12} + 2\,v_5 \wedge 2 \leq$
$v_{12} + 2\,v_6 \wedge 2 \leq v_{12} + 2\,v_7 \wedge 2 \leq v_{12} + 2\,v_8 \wedge 2 \leq v_{12} + 2\,v_9 \wedge 2 \leq 2\,v_{10} + v_{12} \wedge 2 \leq 2\,v_{11} + v_{12} \wedge 2 \leq$
$v_1 + 2\,v_{12} \wedge 2 \leq 2\,v_{12} + v_2 \wedge 2 \leq 2\,v_{12} + v_3 \wedge 2 \leq 2\,v_{12} + v_4 \wedge 2 \leq 2\,v_{12} + v_5 \wedge 2 \leq 2\,v_{12} + v_6 \wedge 2 \leq$
$2\,v_{12} + v_7 \wedge 2 \leq 2\,v_{12} + v_8 \wedge 2 \leq 2\,v_{12} + v_9 \wedge 2 \leq v_{10} + 2\,v_{12} \wedge 2 \leq v_{11} + 2\,v_{12} \wedge v_1 \leq v_{13} \wedge v_2 \leq$
$v_{13} \wedge v_3 \leq v_{13} \wedge v_4 \leq v_{13} \wedge v_5 \leq v_{13} \wedge v_6 \leq v_{13} \wedge v_7 \leq v_{13} \wedge v_8 \leq v_{13} \wedge v_9 \leq v_{13} \wedge v_{10} \leq$
$v_{13} \wedge v_{11} \leq v_{13} \wedge v_{12} \leq v_{13} \wedge v_{14} \leq v_{13} \wedge v_{15} \leq v_{13} \wedge v_{16} \leq v_{13} \wedge v_{17} \leq v_{13} \wedge v_{18} \leq v_{13} \wedge v_{19} \leq$
$v_{13} \wedge v_{20} \leq v_{13} \wedge v_{21} \leq v_{13} \wedge 1 \leq 2\,v_{13} \wedge 2 \leq 2\,v_1 + v_{13} \wedge 2 \leq v_{13} + 2\,v_2 \wedge 2 \leq v_{13} + 2\,v_3 \wedge 2 \leq$
$v_{13} + 2\,v_4 \wedge 2 \leq v_{13} + 2\,v_5 \wedge 2 \leq v_{13} + 2\,v_6 \wedge 2 \leq v_{13} + 2\,v_7 \wedge 2 \leq v_{13} + 2\,v_8 \wedge 2 \leq v_{13} + 2\,v_9 \wedge 2 \leq$
$2\,v_{10} + v_{13} \wedge 2 \leq 2\,v_{11} + v_{13} \wedge 2 \leq 2\,v_{12} + v_{13} \wedge 2 \leq v_1 + 2\,v_{13} \wedge 2 \leq 2\,v_{13} + v_2 \wedge 2 \leq 2\,v_{13} + v_3 \wedge 2 \leq$
$2\,v_{13} + v_4 \wedge 2 \leq 2\,v_{13} + v_5 \wedge 2 \leq 2\,v_{13} + v_6 \wedge 2 \leq 2\,v_{13} + v_7 \wedge 2 \leq 2\,v_{13} + v_8 \wedge 2 \leq 2\,v_{13} + v_9 \wedge 2 \leq$
$v_{10} + 2\,v_{13} \wedge 2 \leq v_{11} + 2\,v_{13} \wedge 2 \leq v_{12} + 2\,v_{13} \wedge v_1 \leq v_{14} \wedge v_2 \leq v_{14} \wedge v_3 \leq v_{14} \wedge v_4 \leq v_{14} \wedge v_5 \leq$
$v_{14} \wedge v_6 \leq v_{14} \wedge v_7 \leq v_{14} \wedge v_8 \leq v_{14} \wedge v_9 \leq v_{14} \wedge v_{10} \leq v_{14} \wedge v_{11} \leq v_{14} \wedge v_{12} \leq v_{14} \wedge v_{13} \leq$
$v_{14} \wedge v_{15} \leq v_{14} \wedge v_{16} \leq v_{14} \wedge v_{17} \leq v_{14} \wedge v_{18} \leq v_{14} \wedge v_{19} \leq v_{14} \wedge v_{20} \leq v_{14} \wedge v_{21} \leq v_{14} \wedge 1 \leq$
$2\,v_{14} \wedge 2 \leq 2\,v_1 + v_{14} \wedge 2 \leq v_{14} + 2\,v_2 \wedge 2 \leq v_{14} + 2\,v_3 \wedge 2 \leq v_{14} + 2\,v_4 \wedge 2 \leq v_{14} + 2\,v_5 \wedge 2 \leq$
$v_{14} + 2\,v_6 \wedge 2 \leq v_{14} + 2\,v_7 \wedge 2 \leq v_{14} + 2\,v_8 \wedge 2 \leq v_{14} + 2\,v_9 \wedge 2 \leq 2\,v_{10} + v_{14} \wedge 2 \leq 2\,v_{11} + v_{14} \wedge 2 \leq$
$2\,v_{12} + v_{14} \wedge 2 \leq 2\,v_{13} + v_{14} \wedge 2 \leq v_1 + 2\,v_{14} \wedge 2 \leq 2\,v_{14} + v_2 \wedge 2 \leq 2\,v_{14} + v_3 \wedge 2 \leq 2\,v_{14} + v_4 \wedge 2 \leq$
$2\,v_{14} + v_5 \wedge 2 \leq 2\,v_{14} + v_6 \wedge 2 \leq 2\,v_{14} + v_7 \wedge 2 \leq 2\,v_{14} + v_8 \wedge 2 \leq 2\,v_{14} + v_9 \wedge 2 \leq v_{10} + 2\,v_{14} \wedge 2 \leq$
$v_{11} + 2\,v_{14} \wedge 2 \leq v_{12} + 2\,v_{14} \wedge 2 \leq v_{13} + 2\,v_{14} \wedge v_1 \leq v_{15} \wedge v_2 \leq v_{15} \wedge v_3 \leq v_{15} \wedge v_4 \leq v_{15} \wedge v_5 \leq$
$v_{15} \wedge v_6 \leq v_{15} \wedge v_7 \leq v_{15} \wedge v_8 \leq v_{15} \wedge v_9 \leq v_{15} \wedge v_{10} \leq v_{15} \wedge v_{11} \leq v_{15} \wedge v_{12} \leq v_{15} \wedge v_{13} \leq$
$v_{15} \wedge v_{14} \leq v_{15} \wedge v_{16} \leq v_{15} \wedge v_{17} \leq v_{15} \wedge v_{18} \leq v_{15} \wedge v_{19} \leq v_{15} \wedge v_{20} \leq v_{15} \wedge v_{21} \leq v_{15} \wedge 1 \leq$
$2\,v_{15} \wedge 2 \leq 2\,v_1 + v_{15} \wedge 2 \leq v_{15} + 2\,v_2 \wedge 2 \leq v_{15} + 2\,v_3 \wedge 2 \leq v_{15} + 2\,v_4 \wedge 2 \leq v_{15} + 2\,v_5 \wedge 2 \leq$
$v_{15} + 2\,v_6 \wedge 2 \leq v_{15} + 2\,v_7 \wedge 2 \leq v_{15} + 2\,v_8 \wedge 2 \leq v_{15} + 2\,v_9 \wedge 2 \leq 2\,v_{10} + v_{15} \wedge 2 \leq 2\,v_{11} + v_{15} \wedge 2 \leq$
$2\,v_{12} + v_{15} \wedge 2 \leq 2\,v_{13} + v_{15} \wedge 2 \leq 2\,v_{14} + v_{15} \wedge 2 \leq v_1 + 2\,v_{15} \wedge 2 \leq 2\,v_{15} + v_2 \wedge 2 \leq 2\,v_{15} + v_3 \wedge 2 \leq$
$2\,v_{15} + v_4 \wedge 2 \leq 2\,v_{15} + v_5 \wedge 2 \leq 2\,v_{15} + v_6 \wedge 2 \leq 2\,v_{15} + v_7 \wedge 2 \leq 2\,v_{15} + v_8 \wedge 2 \leq 2\,v_{15} + v_9 \wedge 2 \leq$
$v_{10} + 2\,v_{15} \wedge 2 \leq v_{11} + 2\,v_{15} \wedge 2 \leq v_{12} + 2\,v_{15} \wedge 2 \leq v_{13} + 2\,v_{15} \wedge 2 \leq v_{14} + 2\,v_{15} \wedge v_1 \leq v_{16} \wedge v_2 \leq$
$v_{16} \wedge v_3 \leq v_{16} \wedge v_4 \leq v_{16} \wedge v_5 \leq v_{16} \wedge v_6 \leq v_{16} \wedge v_7 \leq v_{16} \wedge v_8 \leq v_{16} \wedge v_9 \leq v_{16} \wedge v_{10} \leq$
$v_{16} \wedge v_{11} \leq v_{16} \wedge v_{12} \leq v_{16} \wedge v_{13} \leq v_{16} \wedge v_{14} \leq v_{16} \wedge v_{15} \leq v_{16} \wedge v_{17} \leq v_{16} \wedge v_{18} \leq v_{16} \wedge v_{19} \leq$
$v_{16} \wedge v_{20} \leq v_{16} \wedge v_{21} \leq v_{16} \wedge 1 \leq 2\,v_{16} \wedge 2 \leq 2\,v_1 + v_{16} \wedge 2 \leq v_{16} + 2\,v_2 \wedge 2 \leq v_{16} + 2\,v_3 \wedge 2 \leq$
$v_{16} + 2\,v_4 \wedge 2 \leq v_{16} + 2\,v_5 \wedge 2 \leq v_{16} + 2\,v_6 \wedge 2 \leq v_{16} + 2\,v_7 \wedge 2 \leq v_{16} + 2\,v_8 \wedge 2 \leq v_{16} + 2\,v_9 \wedge 2 \leq$
$2\,v_{10} + v_{16} \wedge 2 \leq 2\,v_{11} + v_{16} \wedge 2 \leq 2\,v_{12} + v_{16} \wedge 2 \leq 2\,v_{13} + v_{16} \wedge 2 \leq 2\,v_{14} + v_{16} \wedge 2 \leq$
$2\,v_{15} + v_{16} \wedge 2 \leq v_1 + 2\,v_{16} \wedge 2 \leq 2\,v_{16} + v_2 \wedge 2 \leq 2\,v_{16} + v_3 \wedge 2 \leq 2\,v_{16} + v_4 \wedge 2 \leq 2\,v_{16} + v_5 \wedge 2 \leq$
$2\,v_{16} + v_6 \wedge 2 \leq 2\,v_{16} + v_7 \wedge 2 \leq 2\,v_{16} + v_8 \wedge 2 \leq 2\,v_{16} + v_9 \wedge 2 \leq v_{10} + 2\,v_{16} \wedge 2 \leq v_{11} + 2\,v_{16} \wedge 2 \leq$
$v_{12} + 2\,v_{16} \wedge 2 \leq v_{13} + 2\,v_{16} \wedge 2 \leq v_{14} + 2\,v_{16} \wedge 2 \leq v_{15} + 2\,v_{16} \wedge v_1 \leq v_{17} \wedge v_2 \leq v_{17} \wedge v_3 \leq$

$v_{17} \wedge v_4 \leq v_{17} \wedge v_5 \leq v_{17} \wedge v_6 \leq v_{17} \wedge v_7 \leq v_{17} \wedge v_8 \leq v_{17} \wedge v_9 \leq v_{17} \wedge v_{10} \leq v_{17} \wedge v_{11} \leq$
$v_{17} \wedge v_{12} \leq v_{17} \wedge v_{13} \leq v_{17} \wedge v_{14} \leq v_{17} \wedge v_{15} \leq v_{17} \wedge v_{16} \leq v_{17} \wedge v_{18} \leq v_{17} \wedge v_{19} \leq v_{17} \wedge v_{20} \leq$
$v_{17} \wedge v_{21} \leq v_{17} \wedge 1 \leq 2\,v_{17} \wedge 2 \leq 2\,v_1 + v_{17} \wedge 2 \leq v_{17} + 2\,v_2 \wedge 2 \leq v_{17} + 2\,v_3 \wedge 2 \leq v_{17} + 2\,v_4 \wedge 2 \leq$
$v_{17} + 2\,v_5 \wedge 2 \leq v_{17} + 2\,v_6 \wedge 2 \leq v_{17} + 2\,v_7 \wedge 2 \leq v_{17} + 2\,v_8 \wedge 2 \leq v_{17} + 2\,v_9 \wedge 2 \leq 2\,v_{10} + v_{17} \wedge 2 \leq$
$2\,v_{11} + v_{17} \wedge 2 \leq 2\,v_{12} + v_{17} \wedge 2 \leq 2\,v_{13} + v_{17} \wedge 2 \leq 2\,v_{14} + v_{17} \wedge 2 \leq 2\,v_{15} + v_{17} \wedge 2 \leq$
$2\,v_{16} + v_{17} \wedge 2 \leq v_1 + 2\,v_{17} \wedge 2 \leq 2\,v_{17} + v_2 \wedge 2 \leq 2\,v_{17} + v_3 \wedge 2 \leq 2\,v_{17} + v_4 \wedge 2 \leq 2\,v_{17} + v_5 \wedge 2 \leq$
$2\,v_{17} + v_6 \wedge 2 \leq 2\,v_{17} + v_7 \wedge 2 \leq 2\,v_{17} + v_8 \wedge 2 \leq 2\,v_{17} + v_9 \wedge 2 \leq v_{10} + 2\,v_{17} \wedge 2 \leq v_{11} + 2\,v_{17} \wedge 2 \leq$
$v_{12} + 2\,v_{17} \wedge 2 \leq v_{13} + 2\,v_{17} \wedge 2 \leq v_{14} + 2\,v_{17} \wedge 2 \leq v_{15} + 2\,v_{17} \wedge 2 \leq v_{16} + 2\,v_{17} \wedge v_1 \leq v_{18} \wedge v_2 \leq$
$v_{18} \wedge v_3 \leq v_{18} \wedge v_4 \leq v_{18} \wedge v_5 \leq v_{18} \wedge v_6 \leq v_{18} \wedge v_7 \leq v_{18} \wedge v_8 \leq v_{18} \wedge v_9 \leq v_{18} \wedge v_{10} \leq$
$v_{18} \wedge v_{11} \leq v_{18} \wedge v_{12} \leq v_{18} \wedge v_{13} \leq v_{18} \wedge v_{14} \leq v_{18} \wedge v_{15} \leq v_{18} \wedge v_{16} \leq v_{18} \wedge v_{17} \leq v_{18} \wedge v_{19} \leq$
$v_{18} \wedge v_{20} \leq v_{18} \wedge v_{21} \leq v_{18} \wedge 1 \leq 2\,v_{18} \wedge 2 \leq 2\,v_1 + v_{18} \wedge 2 \leq v_{18} + 2\,v_2 \wedge 2 \leq v_{18} + 2\,v_3 \wedge 2 \leq$
$v_{18} + 2\,v_4 \wedge 2 \leq v_{18} + 2\,v_5 \wedge 2 \leq v_{18} + 2\,v_6 \wedge 2 \leq v_{18} + 2\,v_7 \wedge 2 \leq v_{18} + 2\,v_8 \wedge 2 \leq v_{18} + 2\,v_9 \wedge 2 \leq$
$2\,v_{10} + v_{18} \wedge 2 \leq 2\,v_{11} + v_{18} \wedge 2 \leq 2\,v_{12} + v_{18} \wedge 2 \leq 2\,v_{13} + v_{18} \wedge 2 \leq 2\,v_{14} + v_{18} \wedge 2 \leq$
$2\,v_{15} + v_{18} \wedge 2 \leq 2\,v_{16} + v_{18} \wedge 2 \leq 2\,v_{17} + v_{18} \wedge 2 \leq v_1 + 2\,v_{18} \wedge 2 \leq 2\,v_{18} + v_2 \wedge 2 \leq 2\,v_{18} + v_3 \wedge 2 \leq$
$2\,v_{18} + v_4 \wedge 2 \leq 2\,v_{18} + v_5 \wedge 2 \leq 2\,v_{18} + v_6 \wedge 2 \leq 2\,v_{18} + v_7 \wedge 2 \leq 2\,v_{18} + v_8 \wedge 2 \leq 2\,v_{18} + v_9 \wedge 2 \leq$
$v_{10} + 2\,v_{18} \wedge 2 \leq v_{11} + 2\,v_{18} \wedge 2 \leq v_{12} + 2\,v_{18} \wedge 2 \leq v_{13} + 2\,v_{18} \wedge 2 \leq v_{14} + 2\,v_{18} \wedge 2 \leq$
$v_{15} + 2\,v_{18} \wedge 2 \leq v_{16} + 2\,v_{18} \wedge 2 \leq v_{17} + 2\,v_{18} \wedge v_1 \leq v_{19} \wedge v_2 \leq v_{19} \wedge v_3 \leq v_{19} \wedge v_4 \leq v_{19} \wedge v_5 \leq$
$v_{19} \wedge v_6 \leq v_{19} \wedge v_7 \leq v_{19} \wedge v_8 \leq v_{19} \wedge v_9 \leq v_{19} \wedge v_{10} \leq v_{19} \wedge v_{11} \leq v_{19} \wedge v_{12} \leq v_{19} \wedge v_{13} \leq$
$v_{19} \wedge v_{14} \leq v_{19} \wedge v_{15} \leq v_{19} \wedge v_{16} \leq v_{19} \wedge v_{17} \leq v_{19} \wedge v_{18} \leq v_{19} \wedge v_{20} \leq v_{19} \wedge v_{21} \leq v_{19} \wedge 1 \leq$
$2\,v_{19} \wedge 2 \leq 2\,v_1 + v_{19} \wedge 2 \leq v_{19} + 2\,v_2 \wedge 2 \leq v_{19} + 2\,v_3 \wedge 2 \leq v_{19} + 2\,v_4 \wedge 2 \leq v_{19} + 2\,v_5 \wedge 2 \leq$
$v_{19} + 2\,v_6 \wedge 2 \leq v_{19} + 2\,v_7 \wedge 2 \leq v_{19} + 2\,v_8 \wedge 2 \leq v_{19} + 2\,v_9 \wedge 2 \leq 2\,v_{10} + v_{19} \wedge 2 \leq 2\,v_{11} + v_{19} \wedge 2 \leq$
$2\,v_{12} + v_{19} \wedge 2 \leq 2\,v_{13} + v_{19} \wedge 2 \leq 2\,v_{14} + v_{19} \wedge 2 \leq 2\,v_{15} + v_{19} \wedge 2 \leq 2\,v_{16} + v_{19} \wedge 2 \leq$
$2\,v_{17} + v_{19} \wedge 2 \leq 2\,v_{18} + v_{19} \wedge 2 \leq v_1 + 2\,v_{19} \wedge 2 \leq 2\,v_{19} + v_2 \wedge 2 \leq 2\,v_{19} + v_3 \wedge 2 \leq 2\,v_{19} + v_4 \wedge 2 \leq$
$2\,v_{19} + v_5 \wedge 2 \leq 2\,v_{19} + v_6 \wedge 2 \leq 2\,v_{19} + v_7 \wedge 2 \leq 2\,v_{19} + v_8 \wedge 2 \leq 2\,v_{19} + v_9 \wedge 2 \leq v_{10} + 2\,v_{19} \wedge 2 \leq$
$v_{11} + 2\,v_{19} \wedge 2 \leq v_{12} + 2\,v_{19} \wedge 2 \leq v_{13} + 2\,v_{19} \wedge 2 \leq v_{14} + 2\,v_{19} \wedge 2 \leq v_{15} + 2\,v_{19} \wedge 2 \leq$
$v_{16} + 2\,v_{19} \wedge 2 \leq v_{17} + 2\,v_{19} \wedge 2 \leq v_{18} + 2\,v_{19} \wedge v_1 \leq v_{20} \wedge v_2 \leq v_{20} \wedge v_3 \leq v_{20} \wedge v_4 \leq v_{20} \wedge v_5 \leq$
$v_{20} \wedge v_6 \leq v_{20} \wedge v_7 \leq v_{20} \wedge v_8 \leq v_{20} \wedge v_9 \leq v_{20} \wedge v_{10} \leq v_{20} \wedge v_{11} \leq v_{20} \wedge v_{12} \leq v_{20} \wedge v_{13} \leq$
$v_{20} \wedge v_{14} \leq v_{20} \wedge v_{15} \leq v_{20} \wedge v_{16} \leq v_{20} \wedge v_{17} \leq v_{20} \wedge v_{18} \leq v_{20} \wedge v_{19} \leq v_{20} \wedge v_{21} \leq v_{20} \wedge 1 \leq$
$2\,v_{20} \wedge 2 \leq 2\,v_1 + v_{20} \wedge 2 \leq 2\,v_2 + v_{20} \wedge 2 \leq v_{20} + 2\,v_3 \wedge 2 \leq v_{20} + 2\,v_4 \wedge 2 \leq v_{20} + 2\,v_5 \wedge 2 \leq$
$v_{20} + 2\,v_6 \wedge 2 \leq v_{20} + 2\,v_7 \wedge 2 \leq v_{20} + 2\,v_8 \wedge 2 \leq v_{20} + 2\,v_9 \wedge 2 \leq 2\,v_{10} + v_{20} \wedge 2 \leq 2\,v_{11} + v_{20} \wedge 2 \leq$
$2\,v_{12} + v_{20} \wedge 2 \leq 2\,v_{13} + v_{20} \wedge 2 \leq 2\,v_{14} + v_{20} \wedge 2 \leq 2\,v_{15} + v_{20} \wedge 2 \leq 2\,v_{16} + v_{20} \wedge 2 \leq$
$2\,v_{17} + v_{20} \wedge 2 \leq 2\,v_{18} + v_{20} \wedge 2 \leq 2\,v_{19} + v_{20} \wedge 2 \leq v_1 + 2\,v_{20} \wedge 2 \leq v_2 + 2\,v_{20} \wedge 2 \leq 2\,v_{20} + v_3 \wedge 2 \leq$
$2\,v_{20} + v_4 \wedge 2 \leq 2\,v_{20} + v_5 \wedge 2 \leq 2\,v_{20} + v_6 \wedge 2 \leq 2\,v_{20} + v_7 \wedge 2 \leq 2\,v_{20} + v_8 \wedge 2 \leq 2\,v_{20} + v_9 \wedge 2 \leq$
$v_{10} + 2\,v_{20} \wedge 2 \leq v_{11} + 2\,v_{20} \wedge 2 \leq v_{12} + 2\,v_{20} \wedge 2 \leq v_{13} + 2\,v_{20} \wedge 2 \leq v_{14} + 2\,v_{20} \wedge 2 \leq$
$v_{15} + 2\,v_{20} \wedge 2 \leq v_{16} + 2\,v_{20} \wedge 2 \leq v_{17} + 2\,v_{20} \wedge 2 \leq v_{18} + 2\,v_{20} \wedge 2 \leq v_{19} + 2\,v_{20} \wedge v_1 \leq v_{21} \wedge v_2 \leq$
$v_{21} \wedge v_3 \leq v_{21} \wedge v_4 \leq v_{21} \wedge v_5 \leq v_{21} \wedge v_6 \leq v_{21} \wedge v_7 \leq v_{21} \wedge v_8 \leq v_{21} \wedge v_9 \leq v_{21} \wedge v_{10} \leq$
$v_{21} \wedge v_{11} \leq v_{21} \wedge v_{12} \leq v_{21} \wedge v_{13} \leq v_{21} \wedge v_{14} \leq v_{21} \wedge v_{15} \leq v_{21} \wedge v_{16} \leq v_{21} \wedge v_{17} \leq v_{21} \wedge v_{18} \leq$
$v_{21} \wedge v_{19} \leq v_{21} \wedge v_{20} \leq v_{21} \wedge 1 \leq 2\,v_{21} \wedge 2 \leq 2\,v_1 + v_{21} \wedge 2 \leq 2\,v_2 + v_{21} \wedge 2 \leq v_{21} + 2\,v_3 \wedge 2 \leq$
$v_{21} + 2\,v_4 \wedge 2 \leq v_{21} + 2\,v_5 \wedge 2 \leq v_{21} + 2\,v_6 \wedge 2 \leq v_{21} + 2\,v_7 \wedge 2 \leq v_{21} + 2\,v_8 \wedge 2 \leq v_{21} + 2\,v_9 \wedge 2 \leq$
$2\,v_{10} + v_{21} \wedge 2 \leq 2\,v_{11} + v_{21} \wedge 2 \leq 2\,v_{12} + v_{21} \wedge 2 \leq 2\,v_{13} + v_{21} \wedge 2 \leq 2\,v_{14} + v_{21} \wedge 2 \leq$
$2\,v_{15} + v_{21} \wedge 2 \leq 2\,v_{16} + v_{21} \wedge 2 \leq 2\,v_{17} + v_{21} \wedge 2 \leq 2\,v_{18} + v_{21} \wedge 2 \leq 2\,v_{19} + v_{21} \wedge 2 \leq$
$2\,v_{20} + v_{21} \wedge 2 \leq v_1 + 2\,v_{21} \wedge 2 \leq v_2 + 2\,v_{21} \wedge 2 \leq 2\,v_{21} + v_3 \wedge 2 \leq 2\,v_{21} + v_4 \wedge 2 \leq 2\,v_{21} + v_5 \wedge 2 \leq$
$2\,v_{21} + v_6 \wedge 2 \leq 2\,v_{21} + v_7 \wedge 2 \leq 2\,v_{21} + v_8 \wedge 2 \leq 2\,v_{21} + v_9 \wedge 2 \leq v_{10} + 2\,v_{21} \wedge 2 \leq v_{11} + 2\,v_{21} \wedge 2 \leq$

$v_{12} + 2\,v_{21} \wedge 2 \leq v_{13} + 2\,v_{21} \wedge 2 \leq v_{14} + 2\,v_{21} \wedge 2 \leq v_{15} + 2\,v_{21} \wedge 2 \leq v_{16} + 2\,v_{21} \wedge 2 \leq$
$v_{17} + 2\,v_{21} \wedge 2 \leq v_{18} + 2\,v_{21} \wedge 2 \leq v_{19} + 2\,v_{21} \wedge 2 \leq v_{20} + 2\,v_{21} \wedge v_{1} \leq 2\,v_{2} \wedge v_{1} \leq 2\,v_{3} \wedge v_{1} \leq$
$2\,v_{4} \wedge v_{1} \leq 2\,v_{5} \wedge v_{1} \leq 2\,v_{6} \wedge v_{1} \leq 2\,v_{7} \wedge v_{1} \leq 2\,v_{8} \wedge v_{1} \leq 2\,v_{9} \wedge v_{1} \leq 2\,v_{10} \wedge v_{1} \leq 2\,v_{11} \wedge v_{1} \leq$
$2\,v_{12} \wedge v_{1} \leq 2\,v_{13} \wedge v_{1} \leq 2\,v_{14} \wedge v_{1} \leq 2\,v_{15} \wedge v_{1} \leq 2\,v_{16} \wedge v_{1} \leq 2\,v_{17} \wedge v_{1} \leq 2\,v_{18} \wedge v_{1} \leq$
$2\,v_{19} \wedge v_{1} \leq 2\,v_{20} \wedge v_{1} \leq 2\,v_{21} \wedge v_{2} \leq 2\,v_{1} \wedge v_{2} \leq 2\,v_{3} \wedge v_{2} \leq 2\,v_{4} \wedge v_{2} \leq 2\,v_{5} \wedge v_{2} \leq 2\,v_{6} \wedge v_{2} \leq$
$2\,v_{7} \wedge v_{2} \leq 2\,v_{8} \wedge v_{2} \leq 2\,v_{9} \wedge v_{2} \leq 2\,v_{10} \wedge v_{2} \leq 2\,v_{11} \wedge v_{2} \leq 2\,v_{12} \wedge v_{2} \leq 2\,v_{13} \wedge v_{2} \leq 2\,v_{14} \wedge v_{2} \leq$
$2\,v_{15} \wedge v_{2} \leq 2\,v_{16} \wedge v_{2} \leq 2\,v_{17} \wedge v_{2} \leq 2\,v_{18} \wedge v_{2} \leq 2\,v_{19} \wedge v_{2} \leq 2\,v_{20} \wedge v_{2} \leq 2\,v_{21} \wedge v_{3} \leq$
$2\,v_{1} \wedge v_{3} \leq 2\,v_{2} \wedge v_{3} \leq 2\,v_{4} \wedge v_{3} \leq 2\,v_{5} \wedge v_{3} \leq 2\,v_{6} \wedge v_{3} \leq 2\,v_{7} \wedge v_{3} \leq 2\,v_{8} \wedge v_{3} \leq 2\,v_{9} \wedge v_{3} \leq$
$2\,v_{10} \wedge v_{3} \leq 2\,v_{11} \wedge v_{3} \leq 2\,v_{12} \wedge v_{3} \leq 2\,v_{13} \wedge v_{3} \leq 2\,v_{14} \wedge v_{3} \leq 2\,v_{15} \wedge v_{3} \leq 2\,v_{16} \wedge v_{3} \leq$
$2\,v_{17} \wedge v_{3} \leq 2\,v_{18} \wedge v_{3} \leq 2\,v_{19} \wedge v_{3} \leq 2\,v_{20} \wedge v_{3} \leq 2\,v_{21} \wedge v_{4} \leq 2\,v_{1} \wedge v_{4} \leq 2\,v_{2} \wedge v_{4} \leq$
$2\,v_{3} \wedge v_{4} \leq 2\,v_{5} \wedge v_{4} \leq 2\,v_{6} \wedge v_{4} \leq 2\,v_{7} \wedge v_{4} \leq 2\,v_{8} \wedge v_{4} \leq 2\,v_{9} \wedge v_{4} \leq 2\,v_{10} \wedge v_{4} \leq 2\,v_{11} \wedge v_{4} \leq$
$2\,v_{12} \wedge v_{4} \leq 2\,v_{13} \wedge v_{4} \leq 2\,v_{14} \wedge v_{4} \leq 2\,v_{15} \wedge v_{4} \leq 2\,v_{16} \wedge v_{4} \leq 2\,v_{17} \wedge v_{4} \leq 2\,v_{18} \wedge v_{4} \leq$
$2\,v_{19} \wedge v_{4} \leq 2\,v_{20} \wedge v_{4} \leq 2\,v_{21} \wedge v_{5} \leq 2\,v_{1} \wedge v_{5} \leq 2\,v_{2} \wedge v_{5} \leq 2\,v_{3} \wedge v_{5} \leq 2\,v_{4} \wedge v_{5} \leq 2\,v_{6} \wedge v_{5} \leq$
$2\,v_{7} \wedge v_{5} \leq 2\,v_{8} \wedge v_{5} \leq 2\,v_{9} \wedge v_{5} \leq 2\,v_{10} \wedge v_{5} \leq 2\,v_{11} \wedge v_{5} \leq 2\,v_{12} \wedge v_{5} \leq 2\,v_{13} \wedge v_{5} \leq 2\,v_{14} \wedge v_{5} \leq$
$2\,v_{15} \wedge v_{5} \leq 2\,v_{16} \wedge v_{5} \leq 2\,v_{17} \wedge v_{5} \leq 2\,v_{18} \wedge v_{5} \leq 2\,v_{19} \wedge v_{5} \leq 2\,v_{20} \wedge v_{5} \leq 2\,v_{21} \wedge v_{6} \leq$
$2\,v_{1} \wedge v_{6} \leq 2\,v_{2} \wedge v_{6} \leq 2\,v_{3} \wedge v_{6} \leq 2\,v_{4} \wedge v_{6} \leq 2\,v_{5} \wedge v_{6} \leq 2\,v_{7} \wedge v_{6} \leq 2\,v_{8} \wedge v_{6} \leq 2\,v_{9} \wedge v_{6} \leq$
$2\,v_{10} \wedge v_{6} \leq 2\,v_{11} \wedge v_{6} \leq 2\,v_{12} \wedge v_{6} \leq 2\,v_{13} \wedge v_{6} \leq 2\,v_{14} \wedge v_{6} \leq 2\,v_{15} \wedge v_{6} \leq 2\,v_{16} \wedge v_{6} \leq$
$2\,v_{17} \wedge v_{6} \leq 2\,v_{18} \wedge v_{6} \leq 2\,v_{19} \wedge v_{6} \leq 2\,v_{20} \wedge v_{6} \leq 2\,v_{21} \wedge v_{7} \leq 2\,v_{1} \wedge v_{7} \leq 2\,v_{2} \wedge v_{7} \leq$
$2\,v_{3} \wedge v_{7} \leq 2\,v_{4} \wedge v_{7} \leq 2\,v_{5} \wedge v_{7} \leq 2\,v_{6} \wedge v_{7} \leq 2\,v_{8} \wedge v_{7} \leq 2\,v_{9} \wedge v_{7} \leq 2\,v_{10} \wedge v_{7} \leq 2\,v_{11} \wedge v_{7} \leq$
$2\,v_{12} \wedge v_{7} \leq 2\,v_{13} \wedge v_{7} \leq 2\,v_{14} \wedge v_{7} \leq 2\,v_{15} \wedge v_{7} \leq 2\,v_{16} \wedge v_{7} \leq 2\,v_{17} \wedge v_{7} \leq 2\,v_{18} \wedge v_{7} \leq$
$2\,v_{19} \wedge v_{7} \leq 2\,v_{20} \wedge v_{7} \leq 2\,v_{21} \wedge v_{8} \leq 2\,v_{1} \wedge v_{8} \leq 2\,v_{2} \wedge v_{8} \leq 2\,v_{3} \wedge v_{8} \leq 2\,v_{4} \wedge v_{8} \leq 2\,v_{5} \wedge v_{8} \leq$
$2\,v_{6} \wedge v_{8} \leq 2\,v_{7} \wedge v_{8} \leq 2\,v_{9} \wedge v_{8} \leq 2\,v_{10} \wedge v_{8} \leq 2\,v_{11} \wedge v_{8} \leq 2\,v_{12} \wedge v_{8} \leq 2\,v_{13} \wedge v_{8} \leq 2\,v_{14} \wedge v_{8} \leq$
$2\,v_{15} \wedge v_{8} \leq 2\,v_{16} \wedge v_{8} \leq 2\,v_{17} \wedge v_{8} \leq 2\,v_{18} \wedge v_{8} \leq 2\,v_{19} \wedge v_{8} \leq 2\,v_{20} \wedge v_{8} \leq 2\,v_{21} \wedge v_{9} \leq$
$2\,v_{1} \wedge v_{9} \leq 2\,v_{2} \wedge v_{9} \leq 2\,v_{3} \wedge v_{9} \leq 2\,v_{4} \wedge v_{9} \leq 2\,v_{5} \wedge v_{9} \leq 2\,v_{6} \wedge v_{9} \leq 2\,v_{7} \wedge v_{9} \leq 2\,v_{8} \wedge v_{9} \leq$
$2\,v_{10} \wedge v_{9} \leq 2\,v_{11} \wedge v_{9} \leq 2\,v_{12} \wedge v_{9} \leq 2\,v_{13} \wedge v_{9} \leq 2\,v_{14} \wedge v_{9} \leq 2\,v_{15} \wedge v_{9} \leq 2\,v_{16} \wedge v_{9} \leq$
$2\,v_{17} \wedge v_{9} \leq 2\,v_{18} \wedge v_{9} \leq 2\,v_{19} \wedge v_{9} \leq 2\,v_{20} \wedge v_{9} \leq 2\,v_{21} \wedge v_{10} \leq 2\,v_{1} \wedge v_{10} \leq 2\,v_{2} \wedge v_{10} \leq$
$2\,v_{3} \wedge v_{10} \leq 2\,v_{4} \wedge v_{10} \leq 2\,v_{5} \wedge v_{10} \leq 2\,v_{6} \wedge v_{10} \leq 2\,v_{7} \wedge v_{10} \leq 2\,v_{8} \wedge v_{10} \leq 2\,v_{9} \wedge v_{10} \leq$
$2\,v_{11} \wedge v_{10} \leq 2\,v_{12} \wedge v_{10} \leq 2\,v_{13} \wedge v_{10} \leq 2\,v_{14} \wedge v_{10} \leq 2\,v_{15} \wedge v_{10} \leq 2\,v_{16} \wedge v_{10} \leq 2\,v_{17} \wedge v_{10} \leq$
$2\,v_{18} \wedge v_{10} \leq 2\,v_{19} \wedge v_{10} \leq 2\,v_{20} \wedge v_{10} \leq 2\,v_{21} \wedge v_{11} \leq 2\,v_{1} \wedge v_{11} \leq 2\,v_{2} \wedge v_{11} \leq 2\,v_{3} \wedge v_{11} \leq$
$2\,v_{4} \wedge v_{11} \leq 2\,v_{5} \wedge v_{11} \leq 2\,v_{6} \wedge v_{11} \leq 2\,v_{7} \wedge v_{11} \leq 2\,v_{8} \wedge v_{11} \leq 2\,v_{9} \wedge v_{11} \leq 2\,v_{10} \wedge v_{11} \leq$
$2\,v_{12} \wedge v_{11} \leq 2\,v_{13} \wedge v_{11} \leq 2\,v_{14} \wedge v_{11} \leq 2\,v_{15} \wedge v_{11} \leq 2\,v_{16} \wedge v_{11} \leq 2\,v_{17} \wedge v_{11} \leq 2\,v_{18} \wedge v_{11} \leq$
$2\,v_{19} \wedge v_{11} \leq 2\,v_{20} \wedge v_{11} \leq 2\,v_{21} \wedge v_{12} \leq 2\,v_{1} \wedge v_{12} \leq 2\,v_{2} \wedge v_{12} \leq 2\,v_{3} \wedge v_{12} \leq 2\,v_{4} \wedge v_{12} \leq$
$2\,v_{5} \wedge v_{12} \leq 2\,v_{6} \wedge v_{12} \leq 2\,v_{7} \wedge v_{12} \leq 2\,v_{8} \wedge v_{12} \leq 2\,v_{9} \wedge v_{12} \leq 2\,v_{10} \wedge v_{12} \leq 2\,v_{11} \wedge v_{12} \leq$
$2\,v_{13} \wedge v_{12} \leq 2\,v_{14} \wedge v_{12} \leq 2\,v_{15} \wedge v_{12} \leq 2\,v_{16} \wedge v_{12} \leq 2\,v_{17} \wedge v_{12} \leq 2\,v_{18} \wedge v_{12} \leq 2\,v_{19} \wedge v_{12} \leq$
$2\,v_{20} \wedge v_{12} \leq 2\,v_{21} \wedge v_{13} \leq 2\,v_{1} \wedge v_{13} \leq 2\,v_{2} \wedge v_{13} \leq 2\,v_{3} \wedge v_{13} \leq 2\,v_{4} \wedge v_{13} \leq 2\,v_{5} \wedge v_{13} \leq$
$2\,v_{6} \wedge v_{13} \leq 2\,v_{7} \wedge v_{13} \leq 2\,v_{8} \wedge v_{13} \leq 2\,v_{9} \wedge v_{13} \leq 2\,v_{10} \wedge v_{13} \leq 2\,v_{11} \wedge v_{13} \leq 2\,v_{12} \wedge v_{13} \leq$
$2\,v_{14} \wedge v_{13} \leq 2\,v_{15} \wedge v_{13} \leq 2\,v_{16} \wedge v_{13} \leq 2\,v_{17} \wedge v_{13} \leq 2\,v_{18} \wedge v_{13} \leq 2\,v_{19} \wedge v_{13} \leq 2\,v_{20} \wedge v_{13} \leq$
$2\,v_{21} \wedge v_{14} \leq 2\,v_{1} \wedge v_{14} \leq 2\,v_{2} \wedge v_{14} \leq 2\,v_{3} \wedge v_{14} \leq 2\,v_{4} \wedge v_{14} \leq 2\,v_{5} \wedge v_{14} \leq 2\,v_{6} \wedge v_{14} \leq$
$2\,v_{7} \wedge v_{14} \leq 2\,v_{8} \wedge v_{14} \leq 2\,v_{9} \wedge v_{14} \leq 2\,v_{10} \wedge v_{14} \leq 2\,v_{11} \wedge v_{14} \leq 2\,v_{12} \wedge v_{14} \leq 2\,v_{13} \wedge v_{14} \leq$
$2\,v_{15} \wedge v_{14} \leq 2\,v_{16} \wedge v_{14} \leq 2\,v_{17} \wedge v_{14} \leq 2\,v_{18} \wedge v_{14} \leq 2\,v_{19} \wedge v_{14} \leq 2\,v_{20} \wedge v_{14} \leq 2\,v_{21} \wedge v_{15} \leq$
$2\,v_{1} \wedge v_{15} \leq 2\,v_{2} \wedge v_{15} \leq 2\,v_{3} \wedge v_{15} \leq 2\,v_{4} \wedge v_{15} \leq 2\,v_{5} \wedge v_{15} \leq 2\,v_{6} \wedge v_{15} \leq 2\,v_{7} \wedge v_{15} \leq$
$2\,v_{8} \wedge v_{15} \leq 2\,v_{9} \wedge v_{15} \leq 2\,v_{10} \wedge v_{15} \leq 2\,v_{11} \wedge v_{15} \leq 2\,v_{12} \wedge v_{15} \leq 2\,v_{13} \wedge v_{15} \leq 2\,v_{14} \wedge v_{15} \leq$
$2\,v_{16} \wedge v_{15} \leq 2\,v_{17} \wedge v_{15} \leq 2\,v_{18} \wedge v_{15} \leq 2\,v_{19} \wedge v_{15} \leq 2\,v_{20} \wedge v_{15} \leq 2\,v_{21} \wedge v_{16} \leq 2\,v_{1} \wedge v_{16} \leq$

$$
\begin{gathered}
2\,v_2 \wedge v_{16} \leq 2\,v_3 \wedge v_{16} \leq 2\,v_4 \wedge v_{16} \leq 2\,v_5 \wedge v_{16} \leq 2\,v_6 \wedge v_{16} \leq 2\,v_7 \wedge v_{16} \leq 2\,v_8 \wedge v_{16} \leq \\
2\,v_9 \wedge v_{16} \leq 2\,v_{10} \wedge v_{16} \leq 2\,v_{11} \wedge v_{16} \leq 2\,v_{12} \wedge v_{16} \leq 2\,v_{13} \wedge v_{16} \leq 2\,v_{14} \wedge v_{16} \leq 2\,v_{15} \wedge v_{16} \leq \\
2\,v_{17} \wedge v_{16} \leq 2\,v_{18} \wedge v_{16} \leq 2\,v_{19} \wedge v_{16} \leq 2\,v_{20} \wedge v_{16} \leq 2\,v_{21} \wedge v_{17} \leq 2\,v_1 \wedge v_{17} \leq 2\,v_2 \wedge v_{17} \leq \\
2\,v_3 \wedge v_{17} \leq 2\,v_4 \wedge v_{17} \leq 2\,v_5 \wedge v_{17} \leq 2\,v_6 \wedge v_{17} \leq 2\,v_7 \wedge v_{17} \leq 2\,v_8 \wedge v_{17} \leq 2\,v_9 \wedge v_{17} \leq \\
2\,v_{10} \wedge v_{17} \leq 2\,v_{11} \wedge v_{17} \leq 2\,v_{12} \wedge v_{17} \leq 2\,v_{13} \wedge v_{17} \leq 2\,v_{14} \wedge v_{17} \leq 2\,v_{15} \wedge v_{17} \leq 2\,v_{16} \wedge v_{17} \leq \\
2\,v_{18} \wedge v_{17} \leq 2\,v_{19} \wedge v_{17} \leq 2\,v_{20} \wedge v_{17} \leq 2\,v_{21} \wedge v_{18} \leq 2\,v_1 \wedge v_{18} \leq 2\,v_2 \wedge v_{18} \leq 2\,v_3 \wedge v_{18} \leq \\
2\,v_4 \wedge v_{18} \leq 2\,v_5 \wedge v_{18} \leq 2\,v_6 \wedge v_{18} \leq 2\,v_7 \wedge v_{18} \leq 2\,v_8 \wedge v_{18} \leq 2\,v_9 \wedge v_{18} \leq 2\,v_{10} \wedge v_{18} \leq \\
2\,v_{11} \wedge v_{18} \leq 2\,v_{12} \wedge v_{18} \leq 2\,v_{13} \wedge v_{18} \leq 2\,v_{14} \wedge v_{18} \leq 2\,v_{15} \wedge v_{18} \leq 2\,v_{16} \wedge v_{18} \leq 2\,v_{17} \wedge v_{18} \leq \\
2\,v_{19} \wedge v_{18} \leq 2\,v_{20} \wedge v_{18} \leq 2\,v_{21} \wedge v_{19} \leq 2\,v_1 \wedge v_{19} \leq 2\,v_2 \wedge v_{19} \leq 2\,v_3 \wedge v_{19} \leq 2\,v_4 \wedge v_{19} \leq \\
2\,v_5 \wedge v_{19} \leq 2\,v_6 \wedge v_{19} \leq 2\,v_7 \wedge v_{19} \leq 2\,v_8 \wedge v_{19} \leq 2\,v_9 \wedge v_{19} \leq 2\,v_{10} \wedge v_{19} \leq 2\,v_{11} \wedge v_{19} \leq \\
2\,v_{12} \wedge v_{19} \leq 2\,v_{13} \wedge v_{19} \leq 2\,v_{14} \wedge v_{19} \leq 2\,v_{15} \wedge v_{19} \leq 2\,v_{16} \wedge v_{19} \leq 2\,v_{17} \wedge v_{19} \leq 2\,v_{18} \wedge v_{19} \leq \\
2\,v_{20} \wedge v_{19} \leq 2\,v_{21} \wedge v_{20} \leq 2\,v_1 \wedge v_{20} \leq 2\,v_2 \wedge v_{20} \leq 2\,v_3 \wedge v_{20} \leq 2\,v_4 \wedge v_{20} \leq 2\,v_5 \wedge v_{20} \leq \\
2\,v_6 \wedge v_{20} \leq 2\,v_7 \wedge v_{20} \leq 2\,v_8 \wedge v_{20} \leq 2\,v_9 \wedge v_{20} \leq 2\,v_{10} \wedge v_{20} \leq 2\,v_{11} \wedge v_{20} \leq 2\,v_{12} \wedge v_{20} \leq \\
2\,v_{13} \wedge v_{20} \leq 2\,v_{14} \wedge v_{20} \leq 2\,v_{15} \wedge v_{20} \leq 2\,v_{16} \wedge v_{20} \leq 2\,v_{17} \wedge v_{20} \leq 2\,v_{18} \wedge v_{20} \leq 2\,v_{19} \wedge v_{20} \leq \\
2\,v_{21} \wedge v_{21} \leq 2\,v_1 \wedge v_{21} \leq 2\,v_2 \wedge v_{21} \leq 2\,v_3 \wedge v_{21} \leq 2\,v_4 \wedge v_{21} \leq 2\,v_5 \wedge v_{21} \leq 2\,v_6 \wedge v_{21} \leq \\
2\,v_7 \wedge v_{21} \leq 2\,v_8 \wedge v_{21} \leq 2\,v_9 \wedge v_{21} \leq 2\,v_{10} \wedge v_{21} \leq 2\,v_{11} \wedge v_{21} \leq 2\,v_{12} \wedge v_{21} \leq 2\,v_{13} \wedge v_{21} \leq \\
2\,v_{14} \wedge v_{21} \leq 2\,v_{15} \wedge v_{21} \leq 2\,v_{16} \wedge v_{21} \leq 2\,v_{17} \wedge v_{21} \leq 2\,v_{18} \wedge v_{21} \leq 2\,v_{19} \wedge v_{21} \leq 2\,v_{20}
\end{gathered}
$$

4.2 0059

Assumptions:

$$
\begin{gathered}
v_1 < v_2 \wedge v_2 < v_3 \wedge v_3 < v_4 \wedge v_4 < v_5 \wedge v_5 < v_6 \wedge v_6 < v_7 \wedge v_7 < v_8 \wedge v_8 < v_9 \wedge v_9 < \\
v_{10} \wedge v_{10} < v_{11} \wedge v_{11} < v_{12} \wedge v_{12} < v_{13} \wedge v_{13} < v_{14} \wedge v_{14} < v_{15} \wedge v_{15} < v_{16} \wedge v_{16} < v_{17} \wedge v_{17} < \\
v_{18} \wedge v_{18} < v_{19} \wedge v_{19} < v_{20} \wedge v_{20} < v_{21} \wedge v_{21} < v_{22} \wedge v_{22} < v_{23} \wedge v_{23} < v_{24} \wedge v_{24} < v_{25} \wedge v_{25} < \\
v_{26} \wedge v_{26} < v_{27} \wedge v_{27} < v_{28} \wedge v_{28} < v_{29} \wedge v_{29} < v_{30} \wedge v_{30} < v_{31} \wedge v_{31} < v_{32} \wedge v_{32} < v_{33} \wedge v_{33} < \\
v_{34} \wedge v_{34} < v_{35} \wedge v_{35} < v_{36} \wedge v_{36} < v_{37} \wedge v_{37} < v_{38} \wedge v_{38} < v_{39} \wedge v_{39} < v_{40} \wedge v_{40} < v_{41}
\end{gathered}
$$

Hypotheses:

$$v_1 < v_{41}$$

4.3 Capital taxes in the NGM 0051

Assumptions:

$$
\begin{gathered}
v_{21} - v_5 = v_4 \wedge v_{21}\,v_7 - v_{21} + v_5 + v_6 = 0 \wedge v_7 - 1 \neq 0 \wedge v_{19}\,v_3 + v_2\,v_{21} = \\
v_{12} \wedge v_{10}\,v_{21} - v_{10}\,v_5 + v_{11}\,v_{19} - v_9 = 0 \wedge -v_2\,v_{22} - v_{20}\,v_3 = 0 \wedge v_3 \neq 0 \wedge -v_{10}\,{v_{15}}^2 v_{22}\,v_8 - \\
v_{11}\,{v_{15}}^2 v_{20}\,v_8 + v_{10}\,{v_{15}}^2 v_{22} + v_{11}\,{v_{15}}^2 v_{20} - v_{11}\,v_{13}\,v_{16} + v_{11}\,v_{14}\,v_{15} - v_{13}\,v_{17}\,v_9 + v_{15}\,v_{16}\,v_9 = \\
0 \wedge {v_{15}}^2 \neq 0 \wedge v_2\,v_{23} + v_{22}\,v_3 = 0 \wedge v_2 \neq 0 \wedge v_{10}\,v_{23}\,{v_7}^2 + v_{11}\,v_{22}\,{v_7}^2 - 2\,v_{10}\,v_{23}\,v_7 - \\
2\,v_{11}\,v_{22}\,v_7 + v_{10}\,v_{23} + v_{11}\,v_{22} - v_5 - v_6 = 0 \wedge (v_7 - 1)^2 \neq 0 \wedge 0 < v_1 \wedge 0 < v_2 \wedge 0 < v_3 \wedge 0 < \\
v_6 \wedge 0 < v_{15} \wedge 0 < -\left(-v_{13}\,v_{16} + v_{14}\,v_{15}\right) {v_{15}}^2 \wedge 0 < -\left(-v_{13}\,v_{17} + v_{15}\,v_{16}\right) {v_{15}}^2 \wedge 0 < \\
v_{18} \wedge 0 < v_{19} \wedge 0 < v_{21} \wedge 0 \leq v_4 \wedge 0 \leq v_5 \wedge 0 \leq v_{22} \wedge v_7 < 1 \wedge v_8 < 1 \wedge v_{13} < 0 \wedge v_{14} < \\
0 \wedge v_{17} < 0 \wedge v_{20} < 0 \wedge v_{14} + v_{16}\,v_{19}\,\left(1 - v_8\right) + v_{15}\,v_{20}\,\left(1 - v_8\right) < 0 \wedge v_{23} < 0
\end{gathered}
$$

Hypotheses:

$$v_{10} < 0$$

4.4 Supply-Demand Scenarios 0012

Assumptions:

$$v_2 = 0 \wedge v_4 = v_3 \wedge v_9 = 1 \wedge v_{10} = 1 \wedge v_1\, v_9 + v_{11}\, v_5 = v_7 \wedge v_{11}\, v_6 + v_2\, v_9 = v_8 \wedge v_{10}\, v_3 + v_{12}\, v_5 = v_7 \wedge v_{10}\, v_4 + v_{12}\, v_6 = v_8 \wedge v_{12} < 0 \wedge v_7 < 0 \wedge v_{11} < 0$$

Hypotheses:

$$0 \le v_5 \vee v_7 < v_8$$

4.5 NGM: Slope of the stable manifold 0026

Assumptions:

$$v_{11} = 0 \wedge v_2\, v_{22} + v_{20}\, v_3 = v_{10} \wedge 0 < v_1 \wedge 0 < v_2 \wedge 0 < v_3 \wedge 0 < v_5 \wedge 0 < v_{14} \wedge 0 < v_{19} \wedge 0 < v_{20} \wedge 0 < v_{22} \wedge 0 \le v_4 \wedge 0 \le v_{23} \wedge v_6 < 1 \wedge v_7 < 1 \wedge v_{12} < 0 \wedge v_{13} < 0 \wedge v_{14} < 0 \wedge v_{21} < 0 \wedge v_{24} < 0 \wedge v_{14}\, v_{22}\, v_6 - v_{11}\, v_{16} - v_{14}\, v_{22} + v_{14}\, v_4 + v_{14}\, v_5 = 0 \wedge v_{16} \ne 0 \wedge v_2\, v_{23} - v_{21}\, v_3 = 0 \wedge v_3 \ne 0 \wedge -{v_{14}}^2 v_{21}\, v_7\, v_9 + {v_{14}}^2 v_{21}\, v_9 - {v_{14}}^2 v_{23}\, v_7 - v_{12}\, v_{15}\, v_9 - v_{12}\, v_{16}\, v_8 + v_{13}\, v_{14}\, v_9 + {v_{14}}^2 v_{23} + v_{14}\, v_{15}\, v_8 = 0 \wedge {v_{14}}^2 \ne 0 \wedge v_2\, v_{24} + v_3\, v\_{23} = 0 \wedge v_2 \ne 0 \wedge -v_{14}\, v_{17}\, v_{22}\, v_6\, v_9 - v_{14}\, v_{18}\, v_{22}\, v_6\, v_8 + v_{15}\, v_{16}\, v_{22}\, v_6\, v_9 + {v_{16}}^2 v_{22}\, v_6\, v_8 - v_{14}\, (1 - v_6)\, v_{16}\, v_{23}\, v_9 + v_{14}\, v_{17}\, v_{22}\, v_9 - v_{14}\, v_{17}\, v_4\, v_9 - v_{14}\, v_{17}\, v_5\, v_9 + v_{14}\, v_{18}\, v_{22}\, v_8 - v_{14}\, v_{18}\, v_4\, v_8 - v_{14}\, v_{18}\, v_5\, v_8 - v_{15}\, v_{16}\, v_{22}\, v_9 - v_{15}\, v_{16}\, v_4\, v_9 + v_{15}\, v_{16}\, v_5\, v_9 - {v_{16}}^2 v_{22}\, v_8 - {v_{16}}^2 v_4\, v_8 + {v_{16}}^2 v_5\, v_8 - v_{14}\, (1 - v_6)\, v_{16}\, v_{24} = 0 \wedge {v_{16}}^2 \ne 0 \wedge 0 < -(-v_{12}\, v_{15} + v_{13}\, v_{14})\, {v_{14}}^2 \wedge 0 < -(-v_{12}\, v_{16} + v_{14}\, v_{15})\, {v_{14}}^2$$

Hypotheses:

$$v_8 < 0$$

4.6 Vector mode: Variance of a sum 0076

Assumptions:

$$v_9 = v_7 + v_8 \wedge 0 \le (-v_7 + x)\,(-v_8 + y)$$

Hypotheses:

$$(-v_7 + x)^2 \le (-v_9 + x + y)^2$$

4.7 Vector mode: Variance of a sum 0077

Assumptions:

$$v_9 = v_7 + v_8 \wedge 1 \le v_1 \wedge 0 \le v_4 \wedge {v_2}^2 \le v_1\, v_4 \wedge 0 \le v_6 \wedge {v_3}^2 \le v_1\, v_6 \wedge {v_5}^2 \le v_4\, v_6 \wedge (-v_1\, v_7\, v_8 + v_2\, v_8 + v_3\, v_7 - v_5)\, v_1 < 0 \vee -v_1\, v_7\, v_8 + v_2\, v_8 + v_3\, v_7 - v_5 = 0 \wedge v_1 \ne 0$$

Hypotheses:

$$(v_1\, {v_7}^2 - v_1\, {v_9}^2 - 2\, v_2\, v_7 + 2\, v_2\, v_9 + 2\, v_3\, v_9 - 2\, v_5 - v_6)\, v_1 < 0 \vee v_1\, {v_7}^2 - v_1\, {v_9}^2 - 2\, v_2\, v_7 + 2\, v_2\, v_9 + 2\, v_3\, v_9 - 2\, v_5 - v_6 = 0 \wedge v_1 \ne 0$$

4.8 A semialgebraic economy's Laffer curve 0057

Assumptions:

$$v_7^2 = v_1^3 \wedge v_8^2 = v_2^3 \wedge 27\, v_{13}^3 v_7 = 8 \wedge 27\, v_{14}^3 v_8 = 8 \wedge v_{13}\ (1 - v_{15}) = v_5 \wedge v_{13}\ (1 - v_{15})\, (v_7 - v_8) = v_1 - v_4 \wedge v_{14}\ (1 - v_{16}) = v_6 \wedge v_{14}\ (1 - v_{16})\, (-v_7 + v_8) = v_2 - v_3 \wedge v_{14}\, v_{16}\, v_8 = v_{13}\, v_{15}\, v_7 \wedge 0 < v_1 \wedge 0 < v_2 \wedge 0 < v_7 \wedge 0 < v_8 \wedge v_{12} < v_9 \wedge v_{11} < v_{10} \wedge 0 < (-v_1 + v_3)\, (v_{11} - v_9) \wedge 0 < (-v_{10} + v_{12})\, (-v_2 + v_4) \wedge 0 < v_{13} \wedge 0 < v_{14} \wedge 0 < v_{15} \wedge v_{15} < v_{16} \wedge v_{16} < 1 \wedge v_8 \neq v_7$$

Hypotheses:

$$v_{10} < v_9$$

4.9 Industry Equilibrium: Marshall's Law 0065

Assumptions:

$$-v_3\, v_5\, v_8 + v_1\, v_4 = 0 \wedge v_3\, v_8 \neq 0 \wedge -v_3\, v_6\, v_8 + v_2\, v_7 = 0 \wedge v_3\, v_8 \neq 0 \wedge v_{12} = 0 \wedge v_{12}\, v_{16} - v_{13}\, v_{16} - v_{10} + v_9 = 0 \wedge v_{12} - v_{13} \neq 0 \wedge v_{17}\, v_7 = v_3 \wedge v_{18} = v_8 \wedge -v_1\, v_9 + v_{19} = 0 \wedge v_1 \neq 0 \wedge -v_{10}\, v_2 + v_{20} = 0 \wedge v_2 \neq 0 \wedge -v_{11}\, v_3 + v_{21} = 0 \wedge v_3 \neq 0 \wedge -v_{12}\, v_4 + v_{22} = 0 \wedge v_4 \neq 0 \wedge -v_{13}\, v_7 + v_{23} = 0 \wedge v_7 \neq 0 \wedge -v_{14}\, v_8 + v_{24} = 0 \wedge v_8 \neq 0 \wedge v_{26}\, v_8 = v_1 \wedge v_{17}\, v_7\, v_8 - v_{26}\, v_4\, v_8 - v_2\, v_7 = 0 \wedge v_7 \neq 0 \wedge v_{17}\, v_{23}\, v_7 + v_{22}\, v_{26}\, v_7 - v_{23}\, v_{26}\, v_4 - v_{21}\, v_7 = 0 \wedge v_7 \neq 0 \wedge -v_{15}\, v_{18} + v_{27}\, v_3 = 0 \wedge v_{18} \neq 0 \wedge v_{21}\, v_{27} = v_{24} \wedge v_1\, v_{19}\, v_{25} + v_1\, v_{20}\, v_{28} - v_{19}\, v_2\, v_{28} - v_1\, v_{24} = 0 \wedge v_1 \neq 0 \wedge v_{22}\, v_{29}\, v_7\, v_8 - v_{23}\, v_{29}\, v_4\, v_8 + v_{24}\, v_{26}\, v_7^2 - v_{19}\, v_7^2 = 0 \wedge v_7^2 \neq 0 \wedge v_{17}\, v_{24}\, v_7^3 - v_{22}\, v_{29}\, v_4\, v_7\, v_8 + v_{23}\, v_{29}\, v_4^2 v_8 - v_{24}\, v_{26}\, v_4\, v_7^2 - v_{20}\, v_7^3 = 0 \wedge v_7^3 \neq 0$$

Hypotheses:

$$v_{13}\ (v_{15}\, v_6 - v_{16}\, v_5) = v_{10}$$

4.10 NGM: Permanent government purchases 0055

Assumptions:

$$v_3 = 1 \wedge v_4 = 1 \wedge v_5 = 0 \wedge v_9 = 0 \wedge v_{12}\, v_7 + v_{14}\, v_5 = v_1 + v_3 - v_5 + v_6 \wedge v_{13}\, v_8 + v_{15}\, v_6 = v_2 + v_4 - v_6 \wedge -v_1\, v_{27}\, v_{31}^2 + v_{15}\, v_2\, v_{28}\, v_{31} + v_{15}\, v_{26}\, v_{31}\, v_8 + v_{18}\, v_{24}\, v_{31}\, v_6 - v_{25}\, v_{31}^2 v_7 - 2\, v_1\, v_{27}\, v_{31} + v_{15}\, v_2\, v_{28} - v_{15}\, v_{24}\, v_9 + v_{15}\, v_{26}\, v_8 + v_{18}\, v_{24}\, v_6 + v_2\, v_{28}\, v_{31} - 2\, v_{25}\, v_{31}\, v_7 + v_{26}\, v_{31}\, v_8 - v_1\, v_{27} + v_2\, v_{28} - v_{24}\, v_9 - v_{25}\, v_7 + v_{26}\, v_8 = 0 \wedge (1 + v_{31})^2 \neq 0 \wedge -v_{16}\, v_{23}^2 v_{32}\, v_7 - v_{10}\, v_{12}\, v_{23}^2 + v_{16}\, v_{23}^2 v_7 - v_1\, v_{19}\, v_{27} + v_1\, v_{23}\, v_{25} - v_{19}\, v_{25}\, v_7 + v_{21}\, v_{23}\, v_7 = 0 \wedge v_{23}^2 \neq 0 \wedge v_{16}\, v_{29}\, v_{32}\, v_7 + v_{10}\, v_{12}\, v_{29} + v_{12}\, v_{32}\, v_7 = v_3 \wedge -v_{17}\, v_{24}^2 v_{33}\, v_8 - v_{11}\, v_{13}\, v_{24}^2 + v_{17}\, v_{24}^2 v_8 - v_2\, v_{20}\, v_{28} + v_2\, v_{24}\, v_{26} - v_{20}\, v_{26}\, v_8 + v_{22}\, v_{24}\, v_8 = 0 \wedge v_{24}^2 \neq 0 \wedge v_{17}\, v_{30}\, v_{33}\, v_8 + v_{11}\, v_{13}\, v_{30} + v_{13}\, v_{33}\, v_8 = v_4 \wedge 0 < v_{12} \wedge 0 < v_{13} \wedge 0 < v_{15} \wedge 0 < v_{23} \wedge 0 < v_{24} \wedge 0 < -(-v_{19}\, v_{25} + v_{21}\, v_{23})\, v_{23}^2 \wedge 0 < -(-v_{20}\, v_{26} + v_{22}\, v_{24})\, v_{24}^2 \wedge 0 < v_{21}\, v_{27} - v_{25}^2 \wedge 0 < -(-v_{19}\, v_{27} + v_{23}\, v_{25})\, v_{23}^2 \wedge 0 < v_{22}\, v_{28} - v_{26}^2 \wedge 0 < -(-v_{20}\, v_{28} + v_{24}\, v_{26})\, v_{24}^2 \wedge 0 < v_{29} \wedge 0 < v_{16}\, v_{29} + v_{12} \wedge 0 < v_{30} \wedge 0 < v_{17}\, v_{30} + v_{13} \wedge -1 < v_{31} \wedge v_{18} < 0 \wedge v_{19} < 0 \wedge v_{20} < 0 \wedge v_{21} < 0 \wedge v_{22} < 0 \wedge v_{27} < 0 \wedge v_{28} < 0 \wedge v_{32} < 1 \wedge v_{33} < 1 \wedge 0 \leq v_{32} \wedge 0 \leq v_{33} \wedge v_{10} \leq 0 \wedge v_{11} \leq 0 \wedge v_{16} \leq 0 \wedge v_{17} \leq 0$$

Hypotheses:

$$0 < v_7$$

4.11 NGM: Temporary government purchases 0053

Assumptions:

$$v_7 = 0 \wedge v_{10} = 0 \wedge v_{12}\, v_9 + v_{13}\, v_7 = 1 + v_6 - v_7 + v_8 \wedge v_{21}\, {v_4}^2 v_9 + v_{22}\, {v_4}^2 v_6 - v_{17}\, v_4\, v_8 + 2\, v_{21}\, v_4\, v_9 + 2\, v_{22}\, v_4\, v_6 + v_{10}\, v_{14} - v_{17}\, v_8 + v_{21}\, v_9 + v_{22}\, v_6 = 0 \wedge (1 + v_4)^2 \neq 0 \wedge -v_{15}\, {v_{20}}^2 v_9 + v_{18}\, v_{21}\, v_9 + v_{18}\, v_{22}\, v_6 - v_{19}\, v_{20}\, v_9 - v_{20}\, v_{21}\, v_6 = 0 \wedge {v_{20}}^2 \neq 0 \wedge 0 < v_1 \wedge 0 < v_3 \wedge -1 < v_4 \wedge 0 < v_{11} \wedge 0 < v_{12} \wedge 0 < v_{13} \wedge 0 < v_{14} \wedge 0 < v_{15}\, v_3 + v_{12} \wedge 0 < v_{20} \wedge 0 < -(-v_{18}\, v_{21} + v_{19}\, v_{20})\, {v_{20}}^2 \wedge 0 < v_{19}\, v_{22} - {v_{21}}^2 \wedge 0 < -(-v_{18}\, v_{22} + v_{20}\, v_{21})\, {v_{20}}^2 \wedge 0 \leq v_2 \wedge v_5 < 1 \wedge v_{16} < 0 \wedge v_{17} < 0 \wedge v_{18} < 0 \wedge v_{19} < 0 \wedge v_{22} < 0 \wedge 0 \leq v_5 \wedge v_{15} \leq 0$$

Hypotheses:

$$0 < v_9 \wedge -1 < v_6 \wedge 0 < v_{12}\, v_9 \wedge v_6 < 0 \wedge v_8 < 0 \wedge v_{12}\, v_9 < 1$$

4.12 NGM: Temporary government purchases 0054

Assumptions:

$$v_8 = 0 \wedge v_{11} = 0 \wedge v_{10}\, v_{12}\, v_4 - v_{10}\, v_{13}\, v_3 + v_{13}\, v_4\, v_8 - v_4\, v_7 + v_4\, v_8 - v_4\, v_9 - v_4 = 0 \wedge v_4 \neq 0 \wedge v_{10}\, v_{20}\, {v_5}^2 + v_{21}\, {v_5}^2 v_7 + 2\, v_{10}\, v_{20}\, v_5 - v_{16}\, v_5\, v_9 + 2\, v_{21}\, v_5\, v_7 + v_{10}\, v_{20} + v_{11}\, v_{14} - v_{16}\, v_9 + v_{21}\, v_7 = 0 \wedge (1 + v_5)^2 \neq 0 \wedge -v_{10}\, v_{15}\, {v_{19}}^2 {v_3}^2 + v_{10}\, v_{17}\, v_{20}\, {v_4}^3 - v_{10}\, v_{18}\, v_{19}\, {v_4}^3 + v_{15}\, {v_{19}}^2 v_3\, v_4\, v_8 + v_{17}\, v_{21}\, {v_4}^3 v_7 - v_{19}\, v_{20}\, {v_4}^3 v_7 = 0 \wedge {v_{19}}^2 {v_4}^3 \neq 0 \wedge 0 < v_1 \wedge 0 < v_3 \wedge 0 < v_4 \wedge -1 < v_5 \wedge 0 < v_{12}\, v_4 \wedge 0 < v_{13} \wedge 0 < -(-v_{12}\, v_4 + v_{13}\, v_3)\, v_4 \wedge 0 < v_{14} \wedge 0 < v_{19} \wedge 0 < -(-v_{17}\, v_{20} + v_{18}\, v_{19})\, {v_{19}}^2 \wedge 0 < v_{18}\, v_{21} - {v_{20}}^2 \wedge 0 < -(-v_{17}\, v_{21} + v_{19}\, v_{20})\, {v_{19}}^2 \wedge 0 \leq v_2 \wedge v_6 < 1 \wedge 0 < -v_{15}\, {v_3}^2 {v_4}^3 \wedge 0 < -v_{15}\, v_4 \wedge v_{16} < 0 \wedge v_{17} < 0 \wedge v_{18} < 0 \wedge v_{21} < 0 \wedge 0 \leq v_6$$

Hypotheses:

$$0 < v_{10} \wedge -1 < v_7 \wedge 0 < -(-v_{10}\, v_{12}\, v_4 + v_{10}\, v_{13}\, v_3 - v_{13}\, v_4\, v_8)\, v_4 \wedge v_7 < 0 \wedge v_9 < 0 \wedge 0 < -v_4\, (v_{10}\, v_{12}\, v_4 - v_{10}\, v_{13}\, v_3 + v_{13}\, v_4\, v_8 - v_4)$$

4.13 NGM: The government purchases multiplier 0027

Assumptions:

$$v_8 = 0 \wedge 0 < v_1 \wedge 0 < v_3 \wedge 0 < v_4 \wedge 0 < v_4\, v_9 \wedge 0 < v_{10} \wedge 0 \leq v_2 \wedge -v_{10}\, v_3\, v_7 + v_{10}\, v_4\, v_6 + v_4\, v_7\, v_9 - v_4\, v_5 - v_4 = 0 \wedge v_4 \neq 0 \wedge -v_{11}\, v_3\, v_7 + v_{11}\, v_4\, v_6 - {v_4}^2 v_8 = 0 \wedge {v_4}^2 \neq 0 \wedge -v_{11}\, {v_{14}}^2 {v_3}^2 v_7 + v_{11}\, {v_{14}}^2 v_3\, v_4\, v_6 + v_{12}\, v_{15}\, {v_4}^3 v_7 + v_{12}\, v_{16}\, {v_4}^3 v_5 - v_{13}\, v_{14}\, {v_4}^3 v_7 - v_{14}\, v_{15}\, {v_4}^3 v_5 = 0 \wedge {v_{14}}^2 {v_4}^3 \neq 0 \wedge 0 < -(v_{10}\, v_3 - v_4\, v_9)\, v_4 \wedge 0 < -(-v_{12}\, v_{15} + v_{13}\, v_{14})\, {v_{14}}^2 \wedge 0 < -(-v_{12}\, v_{16} + v_{14}\, v_{15})\, {v_{14}}^2 \wedge 0 < -v_{11}\, {v_3}^2 {v_4}^3 \wedge 0 < -v_{11}\, v_4$$

Hypotheses:

$$0 < v_6 \wedge 0 < v_7 \wedge -1 < v_5 \wedge 0 < -(v_{10}\, v_3\, v_7 - v_{10}\, v_4\, v_6 - v_4\, v_7\, v_9)\, v_4 \wedge v_5 < 0 \wedge 0 < -(-v_{10}\, v_3\, v_7 + v_{10}\, v_4\, v_6 + v_4\, v_7\, v_9 - v_4)\, v_4$$

4.14 NGM: The government purchases multiplier 0028

Assumptions:

$$v_8 = 0 \wedge v_{11} = 0 \wedge v_{10}\, v_{12}\, v_4 - v_{10}\, v_{13}\, v_3 + v_{13}\, v_4\, v_8 - v_4\, v_7 + v_4\, v_8 - v_4\, v_9 - v_4 = 0 \wedge v_4 \neq 0 \wedge v_{10}\, v_{20}\, {v_5}^2 + v_{21}\, {v_5}^2 v_7 + 2\, v_{10}\, v_{20}\, v_5 - v_{16}\, v_5\, v_9 + 2\, v_{21}\, v_5\, v_7 + v_{10}\, v_{20} + v_{11}\, v_{14} - v_{16}\, v_9 + v_{21}\, v_7 = 0 \wedge (1 + v_5)^2 \neq 0 \wedge -v_{10}\, v_{15}\, {v_{19}}^2 {v_3}^2 + v_{10}\, v_{17}\, v_{20}\, {v_4}^3 - v_{10}\, v_{18}\, v_{19}\, {v_4}^3 + v_{15}\, {v_{19}}^2 v_3\, v_4\, v_8 + v_{17}\, v_{21}\, {v_4}^3 v_7 - v_{19}\, v_{20}\, {v_4}^3 v_7 = 0 \wedge {v_{19}}^2 {v_4}^3 \neq 0 \wedge 0 < v_1 \wedge 0 < v_3 \wedge 0 < v_4 \wedge -1 < v_5 \wedge 0 < v_{12}\, v_4 \wedge 0 < v_{13} \wedge 0 < -(-v_{12}\, v_4 + v_{13}\, v_3)\, v_4 \wedge 0 < v_{14} \wedge 0 < v_{19} \wedge 0 < -(-v_{17}\, v_{20} + v_{18}\, v_{19})\, {v_{19}}^2 \wedge 0 < v_{18}\, v_{21} - {v_{20}}^2 \wedge 0 < -(-v_{17}\, v_{21} + v_{19}\, v_{20})\, {v_{19}}^2 \wedge 0 \leq v_2 \wedge v_6 < 1 \wedge 0 < -v_{15}\, {v_3}^2 {v_4}^3 \wedge 0 < -v_{15}\, v_4 \wedge v_{16} < 0 \wedge v_{17} < 0 \wedge v_{18} < 0 \wedge v_{21} < 0 \wedge 0 \leq v_6$$

Hypotheses:

$$0 < v_{10} \wedge -1 < v_7 \wedge 0 < -(-v_{10}\, v_{12}\, v_4 + v_{10}\, v_{13}\, v_3 - v_{13}\, v_4\, v_8)\, v_4 \wedge v_7 < 0 \wedge v_9 < 0 \wedge 0 < -v_4\, (v_{10}\, v_{12}\, v_4 - v_{10}\, v_{13}\, v_3 + v_{13}\, v_4\, v_8 - v_4)$$

4.15 Supply-Demand: Becker irrational demand 0018

Assumptions:

$$v_2 = v_1 \wedge v_3\, v_5 + v_4\, v_7 = v_1 \wedge v_3\, v_6 + v_4\, v_7 = v_1 \wedge v_3\, v_6 + v_4\, v_8 = v_2 \wedge 0 < v_3 \wedge 0 < v_4 \wedge v_6 < v_5$$

Hypotheses:

$$v7 < v8$$

4.16 Supply-Demand: Determinants of quantity 0001

Assumptions:

$$v_7 < 0 \wedge 0 < v_8 \wedge 0 < v_4 \wedge v_2\, v_6 + v_3\, v_8 = v_4 \wedge v_1\, v_5 + v_3\, v_7 = v_4 \wedge v_6 = 1 \wedge v_5 = 1$$

Hypotheses:

$$0 < v_1$$

4.17 Supply-Demand: Determinants of quantity 0004

Assumptions:

$$v_5 = 1 \wedge v_6 = 1 \wedge v_1\, v_5 + v_3\, v_7 = v_4 \wedge v_2\, v_6 + v_3\, v_8 = v_4 \wedge 0 < v_8 \wedge v_7 < 0$$

Hypotheses:

$$0 < v_1 \vee 0 < v_2 \vee v_4 \leq 0$$

4.18 Supply-Demand: Determinants of quantity 0009

Assumptions:

$$v_5 = 1 \wedge v_6 = 1 \wedge v_1\, v_5 + v_3\, v_7 = v_4 \wedge v_2\, v_6 + v_3\, v_8 = v_4 \wedge 0 < v_8 \wedge v_7 < 0$$

Hypotheses:

$$0 \leq v_4 \vee v_1 < 0 \vee v_2 < 0$$

4.19 Supply-Demand: Global demand analysis 0074

Assumptions:

$$v_5 = v_2 \wedge v_6 = v_3 \wedge 0 < (v_5 - v_6)\,(v_7 - v_8) \wedge v_3 < v_4 \wedge (v_1 - v_3)\,(v_7 - v_8) < 0 \wedge (v_2 - v_4)\,(v_7 - v_8) < 0$$

Hypotheses:

$$v_8 < v_7$$

4.20 Supply-Demand: Krugman scenario error 0013

Assumptions:

$$v_2 = 0 \wedge v_4 = v_3 \wedge v_9 = 1 \wedge v_{10} = 1 \wedge v_1\, v_9 + v_{11}\, v_5 = v_7 \wedge v_{11}\, v_6 + v_2\, v_9 = v_8 \wedge v_{10}\, v_3 + v_{12}\, v_5 = v_7 \wedge v_{10}\, v_4 + v_{12}\, v_6 = v_8 \wedge 0 < v_{12} \wedge v_5 < 0 \wedge v_7 < 0 \wedge v_{11} < 0$$

Hypotheses:

$$v_7 < 2\, v_8 \wedge v_8 < 0$$

4.21 Vector mode: Consumer Theory Adding Up 0080

Assumptions:

$$v_2\, v_5 = 0 \wedge v_1\, v_9 = v_5 \wedge 0 < v_1 \wedge 0 < v_2$$

Hypotheses:

$$v_9 = 0$$

4.22 Industry Equilibrium: Adding up for factor shares 0063

Assumptions:

$$-v_3\, v_5\, v_8 + v_1\, v_4 = 0 \wedge v_3\, v_8 \neq 0 \wedge -v_3\, v_6\, v_8 + v_2\, v_7 = 0 \wedge v_3\, v_8 \neq 0 \wedge v_7\, v_9 = v_3 \wedge v_{10}\, v_8 = v_1 \wedge -v_{10}\, v_4\, v_8 + v_7\, v_8\, v_9 - v_2\, v_7 = 0 \wedge v_7 \neq 0$$

Hypotheses:

$$v_6 = 1 - v_5$$

4.23 Industry Equilibrium: Input and output quantities 0064

Assumptions:

$$-v_3\,v_5\,v_7 + v_1\,v_4 = 0 \wedge v_3\,v_7 \neq 0 \wedge v_{11} = 0 \wedge v_{14}\,v_6 = v_3 \wedge -v_1\,v_8 + v_{15} = 0 \wedge v_1 \neq 0 \wedge -v_2\,v_9 + v_{16} = 0 \wedge v_2 \neq 0 \wedge -v_{10}\,v_3 + v_{17} = 0 \wedge v_3 \neq 0 \wedge -v_{11}\,v_4 + v_{18} = 0 \wedge v_4 \neq 0 \wedge -v_{12}\,v_6 + v_{19} = 0 \wedge v_6 \neq 0 \wedge -v_{13}\,v_7 + v_{20} = 0 \wedge v_7 \neq 0 \wedge v_{22}\,v_7 = v_1 \wedge v_{14}\,v_6\,v_7 - v_{22}\,v_4\,v_7 - v_2\,v_6 = 0 \wedge v_6 \neq 0 \wedge v_1\,v_{15}\,v_{21} + v_1\,v_{16}\,v_{23} - v_{15}\,v_2\,v_{23} - v_1\,v_{20} = 0 \wedge v_1 \neq 0 \wedge v_{18}\,v_{24}\,v_6\,v_7 - v_{19}\,v_{24}\,v_4\,v_7 + v_{20}\,v_{22}\,{v_6}^2 - v_{15}\,{v_6}^2 = 0 \wedge {v_6}^2 \neq 0 \wedge v_{14}\,v_{20}\,{v_6}^3 - v_{18}\,v_{24}\,v_4\,v_6\,v_7 + v_{19}\,v_{24}\,{v_4}^2 v_7 - v_{20}\,v_{22}\,v_4\,{v_6}^2 - v_{16}\,{v_6}^3 = 0 \wedge {v_6}^3 \neq 0$$

Hypotheses:

$$v_{13} = v_5\,(v_8 - v_9) + v_9$$

4.24 Industry Equilibrium: Prices and factor costs 0061

Assumptions:

$$-v_3\,v_5\,v_8 + v_1\,v_4 = 0 \wedge v_3\,v_8 \neq 0 \wedge -v_3\,v_6\,v_8 + v_2\,v_7 = 0 \wedge v_3\,v_8 \neq 0 \wedge -v_3\,v_9 + v_{13} = 0 \wedge v_3 \neq 0 \wedge -v_{10}\,v_4 + v_{14} = 0 \wedge v_4 \neq 0 \wedge -v_{11}\,v_7 + v_{15} = 0 \wedge v_7 \neq 0 \wedge v_{16}\,v_8 = v_1 \wedge v_{12}\,v_7\,v_8 - v_{16}\,v_4\,v_8 - v_2\,v_7 = 0 \wedge v_7 \neq 0 \wedge v_{12}\,v_{15}\,v_7 + v_{14}\,v_{16}\,v_7 - v_{15}\,v_{16}\,v_4 - v_{13}\,v_7 = 0 \wedge v_7 \neq 0$$

Hypotheses:

$$v_1\,v_6 + v_{10}\,v_5 = v_9$$

4.25 Jehle and Reny Theorem 3.1 with three inputs 0056

Assumptions:

$$v_1\,v_4 + v_2\,v_6 + v_3\,v_5 = 0 \wedge v_1\,v_2 + v_{10}\,v_5 + v_6\,v_8 = 0 \wedge v_3\,v_1 + v_{10}\,v_6 + v_{11}\,v_5 = 0 \wedge 0 < v_1 \wedge 0 < v_5 \wedge 0 < v_6 \wedge 0 < v_7 \wedge 0 < v_9 \wedge 0 < v_{12} \wedge v_{11}\,{v_{12}}^2 + v_4\,{v_9}^2 < 2\,v_{12}\,v_3\,v_9 \wedge {v_{10}}^2 {v_{12}}^2 + 2\,v_{11}\,v_{12}\,v_2\,v_7 + {v_2}^2 {v_9}^2 + {v_3}^2 {v_7}^2 < 2\,v_2\,(v_{10}\,v_{12} + v_3\,v_7)\,v_9 + v_{12}\,(2\,v_{10}\,v_3\,v_7 + v_{11}\,v_{12}\,v_8 - 2\,v_3\,v_8\,v_9) + v_4\,(-2\,v_{10}\,v_7\,v_9 + v_{11}\,{v_7}^2 + v_8\,{v_9}^2)$$

Hypotheses:

$${v_2}^2 \leq v_4\,v_8 \wedge {v_3}^2 \leq v_{11}\,v_4 \wedge {v_{10}}^2 \leq v_{11}\,v_8 \wedge 2\,v_{10}\,v_2\,v_3 \leq {v_{10}}^2 v_4 + {v_3}^2 v_8 + v_{11}\,({v_2}^2 - v_4\,v_8) \wedge v_4 \leq 0 \wedge v_8 \leq 0 \wedge v_{11} \leq 0$$

4.26 NGM: Capital taxes reduce steady-state capital 0022

Assumptions:

$$v_{21} - v_5 = v_4 \wedge v_{21}\,v_7 - v_{21} + v_5 + v_6 = 0 \wedge v_7 - 1 \neq 0 \wedge v_{19}\,v_3 + v_2\,v_{21} = v_{12} \wedge v_{10}\,v_{21} - v_{10}\,v_5 + v_{11}\,v_{19} - v_9 = 0 \wedge -v_2\,v_{22} - v_{20}\,v_3 = 0 \wedge v_3 \neq 0 \wedge -v_{10}\,{v_{15}}^2 v_{22}\,v_8 - v_{11}\,{v_{15}}^2 v_{20}\,v_8 + v_{10}\,{v_{15}}^2 v_{22} + v_{11}\,{v_{15}}^2 v_{20} - v_{11}\,v_{13}\,v_{16} + v_{11}\,v_{14}\,v_{15} - v_{13}\,v_{17}\,v_9 - {v_{15}}^2 v_{19} + v_{15}\,v_{16}\,v_9 = 0 \wedge {v_{15}}^2 \neq 0 \wedge v_2\,v_{23} - v_{22}\,v_3 = 0 \wedge v_2 \neq 0 \wedge v_{10}\,v_{23} + v_{11}\,v_{22} = 0 \wedge 0 < v_1 \wedge 0 < v_2 \wedge 0 <$$

$v_3 \wedge 0 < v_6 \wedge 0 < v_{15} \wedge 0 < -(-v_{13}\, v_{16} + v_{14}\, v_{15})\, {v_{15}}^2 \wedge 0 < -(-v_{13}\, v_{17} + v_{15}\, v_{16})\, {v_{15}}^2 \wedge 0 < v_{18} \wedge 0 < v_{19} \wedge 0 < v_{21} \wedge 0 \leq v_4 \wedge 0 \leq v_5 \wedge 0 \leq v_{22} \wedge v_7 < 1 \wedge v_8 < 1 \wedge v_{13} < 0 \wedge v_{14} < 0 \wedge v_{17} < 0 \wedge v_{20} < 0 \wedge v_{14} + v_{16}\, v_{19}\, (1 - v_8) + v_{15}\, v_{20}\, (1 - v_8) < 0 \wedge v_{23} < 0$

Hypotheses:

$$v_{10} < 0$$

4.27 NGM: Capital taxes reduce steady-state consumption 0023

Assumptions:

$v_{21} - v_5 = v_4 \wedge v_{21}\, v_7 - v_{21} + v_5 + v_6 = 0 \wedge v_7 - 1 \neq 0 \wedge v_{19}\, v_3 + v_2\, v_{21} = v_{12} \wedge v_{10}\, v_{21} - v_{10}\, v_5 + v_{11}\, v_{19} - v_9 = 0 \wedge -v_2\, v_{22} - v_{20}\, v_3 = 0 \wedge v_3 \neq 0 \wedge -v_{10}\, {v_{15}}^2 v_{22}\, v_8 - v_{11}\, {v_{15}}^2 v_{20}\, v_8 + v_{10}\, {v_{15}}^2 v_{22} + v_{11}\, {v_{15}}^2 v_{20} - v_{11}\, v_{13}\, v_{16} + v_{11}\, v_{14}\, v_{15} - v_{13}\, v_{17}\, v_9 - {v_{15}}^2 v_{19} + v_{15}\, v_{16}\, v_9 = 0 \wedge {v_{15}}^2 \neq 0 \wedge v_2\, v_{23} - v_{22}\, v_3 = 0 \wedge v_2 \neq 0 \wedge v_{10}\, v_{23} + v_{11}\, v_{22} = 0 \wedge 0 < v_1 \wedge 0 < v_2 \wedge 0 < v_3 \wedge 0 < v_6 \wedge 0 < v_{15} \wedge 0 < -(-v_{13}\, v_{16} + v_{14}\, v_{15})\, {v_{15}}^2 \wedge 0 < -(-v_{13}\, v_{17} + v_{15}\, v_{16})\, {v_{15}}^2 \wedge 0 < v_{18} \wedge 0 < v_{19} \wedge 0 < v_{21} \wedge 0 \leq v_4 \wedge 0 \leq v_5 \wedge 0 \leq v_{22} \wedge v_7 < 1 \wedge v_8 < 1 \wedge v_{13} < 0 \wedge v_{14} < 0 \wedge v_{17} < 0 \wedge v_{20} < 0 \wedge v_{14} + v_{16}\, v_{19}\, (1 - v_8) + v_{15}\, v_{20}\, (1 - v_8) < 0 \wedge v_{23} < 0$

Hypotheses:

$$v_9 < 0$$

4.28 NGM: Labor tax impact with separable production 0024

Assumptions:

$v_{19}\, v_6 - v_{19} + v_4 + v_5 = 0 \wedge v_6 - 1 \neq 0 \wedge 0 < -(-v_{11}\, v_{15} + v_{13}\, v_{14})\, {v_{13}}^2 \wedge 0 < -(-v_{11}\, v_{14} + v_{12}\, v_{13})\, {v_{13}}^2 \wedge -v_{10}\, {v_{13}}^2 v_{18}\, v_7 - {v_{13}}^2 v_{20}\, v_7\, v_9 + v_{10}\, {v_{13}}^2 v_{18} + {v_{13}}^2 v_{20}\, v_9 - v\_8\, v_{11}\, v_{15} - v_{10}\, v_{11}\, v_{14} - v_{10}\, v_{12}\, v_{13} - {v_{13}}^2 v_{17} - v_{13}\, v_{14}\, v_8 = 0 \wedge {v_{13}}^2 \neq 0 \wedge v\_17\, v_{10} + v_{19}\, v_9 - v_4\, v_9 - v_8 = 0 \wedge v_{20} = 0 \wedge v_{10}\, v_{20} + v_{21}\, v_9 = 0 \wedge 0 < v_1 \wedge 0 < v_2 \wedge 0 < v_3 \wedge 0 < v_5 \wedge 0 < v_{13} \wedge 0 < v_{16} \wedge 0 < v_{17} \wedge 0 < v_{19} \wedge 0 \leq v_4 \wedge 0 \leq v_{20} \wedge v_6 < 1 \wedge v_7 < 1 \wedge v_{11} < 0 \wedge v_{12} < 0 \wedge v_{15} < 0 \wedge v_{18} < 0 \wedge v_{12} + v_{14}\, v_{17}\, (1 - v_7) + v_{13}\, v_{18}\, (1 - v_7) < 0 \wedge v_{21} < 0$

Hypotheses:

$$v_9 = 0 \wedge v_{10} < 0$$

4.29 NGM: Labor tax impact with separable production 0025

Assumptions:

$v_{19}\, v_6 - v_{19} + v_4 + v_5 = 0 \wedge v_6 - 1 \neq 0 \wedge 0 < -(-v_{11}\, v_{15} + v_{13}\, v_{14})\, {v_{13}}^2 \wedge 0 < -(-v_{11}\, v_{14} + v_{12}\, v_{13})\, {v_{13}}^2 \wedge -v_{10}\, {v_{13}}^2 v_{18}\, v_7 - {v_{13}}^2 v_{20}\, v_7\, v_9 + v_{10}\, {v_{13}}^2 v_{18} + {v_{13}}^2 v_{20}\, v_9 - v\_8\, v_{11}\, v_{15} - v_{10}\, v_{11}\, v_{14} - v_{10}\, v_{12}\, v_{13} - {v_{13}}^2 v_{17} - v_{13}\, v_{14}\, v_8 = 0 \wedge {v_{13}}^2 \neq 0 \wedge v\_17\, v_{10} + v_{19}\, v_9 - v_4\, v_9 - v_8 = 0 \wedge v_{20} = 0 \wedge v_{10}\, v_{20} + v_{21}\, v_9 = 0 \wedge 0 < v_1 \wedge 0 < v_2 \wedge 0 < v_3 \wedge 0 < v_5 \wedge 0 < v_{13} \wedge 0 < v_{16} \wedge 0 < v_{17} \wedge 0 < v_{19} \wedge 0 \leq v_4 \wedge 0 \leq v_{20} \wedge v_6 < 1 \wedge v_7 < 1 \wedge v_{11} < 0 \wedge v_{12} < 0 \wedge v_{15} < 0 \wedge v_{18} < 0 \wedge v_{12} + v_{14}\, v_{17}\, (1 - v_7) + v_{13}\, v_{18}\, (1 - v_7) < 0 \wedge v_{21} < 0$

Hypotheses:

$$v_8 < 0$$

4.30 NGM: Signing the impact of purchases on labor 0030

Assumptions:

$$v_9 = 0 \wedge v_{11}\, v_4\, v_8 - v_{12}\, v_3\, v_8 + v_{12}\, v_4\, v_7 - v_4\, v_6 - v_4 = 0 \wedge v_4 \neq 0 \wedge -v_{13}\, v_3\, v_8 + v_{13}\, v_4\, v_7 - {v_4}^2 v_9 = 0 \wedge {v_4}^2 \neq 0 \wedge v_{10}\, v_{11}\, {v_4}^3 - v_{10}\, v_{12}\, v_3\, {v_4}^2 + v_{11}\, {v_4}^2 v_5\, v_8 - v_{12}\, v_3\, v_4\, v_5\, v_8 + v_{13}\, {v_3}^2 v_5\, v_8 - v_{13}\, v_3\, v_4\, v_5\, v_7 - {v_4}^2 = 0 \wedge {v_4}^2 \neq 0 \wedge v_{10}\, v_{11}\, {v_{16}}^2 {v_4}^3 - v_{10}\, v_{12}\, {v_{16}}^2 v_3\, {v_4}^2 + v_{13}\, {v_{16}}^2 {v_3}^2 v_5\, v_8 - v_{13}\, {v_{16}}^2 v_3\, v_4\, v_5\, v_7 - v_{13}\, {v_{16}}^2 {v_3}^2 v_8 + v_{13}\, {v_{16}}^2 v_3\, v_4\, v_7 + v_{14}\, v_{17}\, {v_4}^3 v_8 + v_{14}\, v_{18}\, {v_4}^3 v_6 + v_{15}\, v_{16}\, {v_4}^3 v_8 - v_{16}\, v_{17}\, {v_4}^3 v_6 = 0 \wedge {v_{16}}^2 {v_4}^3 \neq 0 \wedge 0 < v_1 \wedge 0 < v_3 \wedge 0 < v_4 \wedge 0 < v_{10} \wedge 0 < v_{11}\, v_4 \wedge 0 < v_{12} \wedge 0 < -\left(-v_{11}\, v_4 + v_{12}\, v_3\right) v_4 \wedge 0 < -\left(-v_{14}\, v_{17} + v_{15}\, v_{16}\right) {v_{16}}^2 \wedge 0 < -\left(-v_{14}\, v_{18} + v_{16}\, v_{17}\right) {v_{16}}^2 \wedge 0 \leq v_2 \wedge v_5 < 1 \wedge 0 < -v_{13}\, {v_3}^2 {v_4}^3 \wedge 0 < -v_{13}\, v_4 \wedge 0 \leq v_5$$

Hypotheses:

$$v_8 = 0 \wedge -v_{10}\, v_{11}\, {v_{16}}^2 v_4 + v_{10}\, v_{12}\, {v_{16}}^2 v_3 + v_{14}\, v_{18}\, v_4 - v_{16}\, v_{17}\, v_4 = 0 \wedge {v_{16}}^2 v_4 \neq 0 \vee 0 < -\left(v_{10}\, v_{11}\, {v_{16}}^2 v_4 - v_{10}\, v_{12}\, {v_{16}}^2 v_3 - v_{14}\, v_{18}\, v_4 + v_{16}\, v_{17}\, v_4\right) v_8\, {v_{16}}^2 v_4$$

4.31 NGM: private and public consumption as substitutes 0052

Assumptions:

$$v_4 = v_3 \wedge v_6 = 0 \wedge v_9 = 0 \wedge v_{10} = 1 \wedge v_{11} = 1 \wedge v_{12} = 0 \wedge v_{13} = 0 \wedge v_{14}\, v_3 - v_{14} - v_4 + 1 = 0 \wedge -1 + v_3 \neq 0 \wedge v_{14} = v_{10} \wedge v_{15} = 0 \wedge v_{16} = 0 \wedge v_{18} = 0 \wedge v_{19} = 0 \wedge v_{20} = 0 \wedge v_{21} = 0 \wedge v_{22} = v_5 \wedge v_{26}\, \left(1 - v_3\right) = v_{23}\, \left(1 - v_4\right) \wedge -v_{10}\, v_{26} + \left(1 - v_3\right) \left(v_{27}\, \left(v_6 - v_8\right) + v_{29}\, v_8\right) = -v_{14}\, v_{23} + \left(1 - v_4\right) \left(v_{24}\, \left(v_6 - v_8\right) + v_{27}\, v_8\right) \wedge -v_{18}\, v_{26} - v_{10}\, \left(v_{27}\, \left(1 - v_7\right) + v_{29}\, v_7\right) + \left(1 - v_3\right) \left(-v_{17}\, v_{27} + v_{17}\, v_{29} + \left(1 - v_7\right) \left(v_{28}\, \left(v_6 - v_8\right) + v_{30}\, v_8\right) + v_7\, \left(v_{30}\, \left(v_6 - v_8\right) + v_{31}\, v_8\right)\right) - \left(v_{27}\, \left(v_6 - v_8\right) + v_{29}\, v_8\right) v_9 = -v_{20}\, v_{23} - v_{14}\, \left(v_{24}\, \left(1 - v_7\right) + v_{27}\, v_7\right) - v_{13}\, \left(v_{24}\, \left(v_6 - v_8\right) + v_{27}\, v_8\right) + \left(1 - v_4\right) \left(-v_{17}\, v_{24} + v_{17}\, v_{27} + \left(1 - v_7\right) \left(v_{25}\, \left(v_6 - v_8\right) + v_{28}\, v_8\right) + v_7\, \left(v_{28}\, \left(v_6 - v_8\right) + v_{30}\, v_8\right)\right) \wedge v_{10}\, v_{34} + v_{14}\, v_{32} + v_{37}\, v_6 = v_{23}\, \left(v_6 - v_8\right) + v_{26}\, v_8 \wedge v_{20}\, v_{32} + v_{18}\, v_{34} + v_{14}\, v_{38} + v_{10}\, v_{39} + v_{40}\, v_6 + v_{13}\, \left(v_{10}\, v_{35} + v_{14}\, v_{33} + v_{38}\, v_6\right) + \left(v_{10}\, v_{36} + v_{14}\, v_{35} + v_{39}\, v_6\right) v_9 = -v_{17}\, v_{23} + v_{17}\, v_{26} + \left(1 - v_7\right) \left(v_{24}\, \left(v_6 - v_8\right) + v_{27}\, v_8\right) + v_7\, \left(v_{27}\, \left(v_6 - v_8\right) + v_{29}\, v_8\right) \wedge v_2 < v_1 \wedge 0 < v_2 \wedge 0 < v_{22} \wedge 0 < v_{23} \wedge 0 < v_{26} \wedge 0 < v_{27} \wedge 0 < v_{24}\, v_{29} - {v_{27}}^2 \wedge v_3 < 1 \wedge v_4 < 1 \wedge v_{24} < 0 \wedge v_{29} < 0$$

Hypotheses:

$$v_{38} + v_{39} = 0$$

4.32 Supply-Demand: The missing Krugman condition 0016

Assumptions:

$$v_2 = 0 \land v_3 = v_1 \land v_4 = v_3 \land v_9 = 1 \land v_{10} = 1 \land v_1\,v_9 + v_{11}\,v_5 = v_7 \land v_{11}\,v_6 + v_2\,v_9 = v_8 \land v_{10}\,v_3 + v_{12}\,v_5 = v_7 \land v_{10}\,v_4 + v_{12}\,v_6 = v_8 \land 0 < v_1 \land 0 < v_{12} \land v_{11} < 0$$

Hypotheses:

$$0 < v7 \land v7 < 2\,v8 \land 2\,v6 < v5 \lor -v11 \leq v12$$

4.33 Vector mode: Constant absolute risk aversion 0082

Assumptions:

$$v_6 = v_3 \land v_1\ ((-1 + v_{10})\,v_3 + (1 - 2\,v_{10})\,v_6 + v_{10}\,v_8) = 0 \land 0 < v_1 \land 2\,v_6 < v_3 + v_8 \land 0 \leq v_4 \land 0 \leq v_7 \land {v_5}^2 \leq v_4\,v_7 \land 0 \leq v_9 \land {v_3}^2 \leq v_2\,v_9 \land {v_6}^2 \leq v_4\,v_9 \land {v_8}^2 \leq v_7\,v_9 \land {v_5}^2 v_9 + {v_6}^2 v_7 \leq 2\,v_5\,v_6\,v_8 + v_4\ (v_7\,v_9 - {v_8}^2)$$

Hypotheses:

$$v_{10} = 0$$

4.34 Vector mode: Constant absolute risk aversion 0083

Assumptions:

$$v_3 - v_6 = 0 \land v_1\,v_2 \neq 0 \land v_{10}\,v_3 - 2\,v_{10}\,v_6 + v_{10}\,v_8 - v_3 + v_6 = 0 \land v_2 \neq 0 \land 0 < v_1 \land 0 < v_4 \land 0 < v_1\,v_2\ (v_3 - 2\,v_6 + v_8) \land 0 < {v_1}^2 v_9 \land 1 \leq v_2 \land 0 \leq v_4 \land 0 \leq v_7 \land {v_5}^2 \leq v_4\,v_7 \land 0 \leq v_9 \land {v_3}^2 \leq v_2\,v_9 \land {v_6}^2 \leq v_4\,v_9 \land {v_8}^2 \leq v_7\,v_9 \land {v_5}^2 v_9 + {v_6}^2 v_7 \leq 2\,v_5\,v_6\,v_8 + v_4\ (v_7\,v_9 - {v_8}^2)$$

Hypotheses:

$$v_{10} = 0$$

4.35 Vector mode: Constant relative risk aversion 0084

Assumptions:

$$v_3\ (v_{10}\,v_2 + v_2\,v_5 - 2\,v_2\,v_8 - v_5 + v_8) = 0 \land v_1\,v_4 \neq 0 \land$$
$$v_3\ (v_1\,v_{10}\,v_{12}\,v_3 + v_1\,v_{12}\,v_3\,v_5 - 2\,v_1\,v_{12}\,v_3\,v_8 + v_1\,v_{10}\,v_2 + v_1\,v_2\,v_5 - 2\,v_1\,v_2\,v_8 - v_1\,v_5 + v_1\,v_8 + v_{10}\,v_2 + v_2\,v_5$$
$$0 \land v_1\,v_4 \neq 0 \land 0 < v_1 \land 0 < v_3 \land 1 \leq v_4 \land 0 \leq v_6 \land 0 \leq v_9 \land {v_7}^2 \leq v_6\,v_9 \land 0 \leq v_{11} \land {v_5}^2 \leq v_{11}\,v_4 \land {v_8}^2 \leq v_{11}\,v_6 \land {v_{10}}^2 \leq v_{11}\,v_9 \land v_{11}\,{v_7}^2 + {v_8}^2 v_9 \leq 2\,v_{10}\,v_7\,v_8 + v_6\ (-{v_{10}}^2 + v_{11}\,v_9) \land {v_3}^2 v_4\ (v_{10} + v_5 - 2\,v_8) < 0 \land 0 \leq v_2 \land v_2 \leq 1$$

Hypotheses:

$$v_{12} = 0$$

4.36 Vector mode: Hicks' generalized law of demand 0078

Assumptions:

$$0 \le v_1 \wedge 0 \le v_{10} \wedge v_4{}^2 \le v_1\, v_{10} \wedge 0 \le v_5 \wedge v_2{}^2 \le v_1\, v_5 \wedge v_7{}^2 \le v_{10}\, v_5 \wedge v_1\, v_7{}^2 + v_{10}\, v_2{}^2 + v_4{}^2 v_5 \le v_1\, v_{10}\, v_5 + 2\, v_2\, v_4\, v_7 \wedge 0 \le v_8 \wedge v_3{}^2 \le v_1\, v_8 \wedge v_9{}^2 \le v_{10}\, v_8 \wedge v_6{}^2 \le v_5\, v_8 \wedge v_1\, v_6{}^2 + v_2{}^2 v_8 + v_3{}^2 v_5 \le v_1\, v_5\, v_8 + 2\, v_2\, v_3\, v_6 \wedge v_1\, v_9{}^2 + v_{10}\, v_3{}^2 + v_4{}^2 v_8 \le v_1\, v_{10}\, v_8 + 2\, v_3\, v_4\, v_9 \wedge v_1\, v_{10}\, v_6{}^2 + v_{10}\, v_2{}^2 v_8 + v_1\, v_7{}^2 v_8 + v_1\, v_5\, v_9{}^2 + 2\, v_4\, \left(v_3\, \left(-v_5\, v_9 + v_6\, v_7\right) + v_2\, \left(v_6\, v_9 - v_7\, v_8\right)\right) \le v_3{}^2 \left(-v_{10}\, v_5 + v_7{}^2\right) + v_1\, v_{10}\, v_5\, v_8 + v_4{}^2 \left(-v_5\, v_8 + v_6{}^2\right) + 2\, v_1\, v_6\, v_7\, v_9 + v_2{}^2 v_9{}^2 + 2\, v_2\, v_3\, \left(v_{10}\, v_6 - v_7\, v_9\right) \wedge v_{10}\, v_6{}^2 + v_7{}^2 v_8 \le 2\, v_6\, v_7\, v_9 + v_5\, \left(v_{10}\, v_8 - v_9{}^2\right) \wedge v_2 \le v_3 \wedge v_9 \le v_7$$

Hypotheses:

$$v_2 + v_9 \le v_3 + v_7$$

4.37 Vector mode: Progressive policy and inequality 0079

Assumptions:

$$v_3\, v_1 < v_5 \wedge 0 \le v_1{}^2 \wedge 0 \le v_4 \wedge 0 \le v_6 \wedge v_3{}^2 \le v_2\, v_6 \wedge v_5{}^2 \le v_4\, v_6$$

Hypotheses:

$$2\, v_5 \le 2\, v_3\, v_1 + v_6$$

4.38 Vector mode: Risk aversion reduces investment 0086

Assumptions:

$$v_{12}\, v_{19} + v_{19}\, v_3 - 2\, v_{19}\, v_8 - v_2 + v_7 = 0 \wedge v_1 \ne 0 \wedge 0 < v_{13} \wedge 1 \le v_1 \wedge 0 \le v_4 \wedge 0 \le v_9 \wedge v_5{}^2 \le v_4\, v_9 \wedge 0 \le v_{13} \wedge v_6{}^2 \le v_{13}\, v_4 \wedge v_{10}{}^2 \le v_{13}\, v_9 \wedge v_{13}\, v_5{}^2 + v_6{}^2 v_9 \le 2\, v_{10}\, v_5\, v_6 + v_4\, \left(-v_{10}{}^2 + v_{13}\, v_9\right) \wedge 0 \le v_{16} \wedge v_2{}^2 \le v_1\, v_{16} \wedge v_7{}^2 \le v_{16}\, v_4 \wedge v_{11}{}^2 \le v_{16}\, v_9 \wedge v_{14}{}^2 \le v_{13}\, v_{16} \wedge v_{16}\, v_5{}^2 + v_7{}^2 v_9 \le 2\, v_{11}\, v_5\, v_7 + v_4\, \left(-v_{11}{}^2 + v_{16}\, v_9\right) \wedge v_{11}{}^2 v_{13}\, v_4 + v_{10}{}^2 v_{16}\, v_4 + v_{13}\, v_{16}\, v_5{}^2 + v_{14}{}^2 v_4\, v_9 + 2\, v_7\, \left(\left(v_{10}\, v_{14} - v_{11}\, v_{13}\right) v_5 + v_6\, \left(v_{10}\, v_{11} - v_{14}\, v_9\right)\right) \le 2\, v_{10}\, v_{11}\, v_{14}\, v_4 + v_{14}{}^2 v_5{}^2 + 2\, \left(v_{10}\, v_{16} - v_{11}\, v_{14}\right) v_5\, v_6 + v_{13}\, v_{16}\, v_4\, v_9 + v_7{}^2 \left(v_{10}{}^2 - v_{13}\, v_9\right) + v_6{}^2 \left(v_{11}{}^2 - v_{16}\, v_9\right) \wedge v_{13}\, v_7{}^2 + v_{16}\, v_6{}^2 \le \left(v_{13}\, v_{16} - v_{14}{}^2\right) v_4 + 2\, v_{14}\, v_6\, v_7 \wedge v_{10}{}^2 v_{16} + v_{11}{}^2 v_{13} \le 2\, v_{10}\, v_{11}\, v_{14} + \left(v_{13}\, v_{16} - v_{14}{}^2\right) v_9 \wedge 0 \le v_{18} \wedge v_3{}^2 \le v_1\, v_{18} \wedge v_8{}^2 \le v_{18}\, v_4 \wedge v_{12}{}^2 \le v_{18}\, v_9 \wedge v_{15}{}^2 \le v_{13}\, v_{18} \wedge v_{17}{}^2 \le v_{16}\, v_{18} \wedge v_{18}\, v_5{}^2 + v_8{}^2 v_9 \le 2\, v_{12}\, v_5\, v_8 + v_4\, \left(-v_{12}{}^2 + v_{18}\, v_9\right) \wedge v_{12}{}^2 v_{13}\, v_4 + v_{10}{}^2 v_{18}\, v_4 + v_{13}\, v_{18}\, v_5{}^2 + v_{15}{}^2 v_4\, v_9 + 2\, v_8\, \left(\left(v_{10}\, v_{15} - v_{12}\, v_{13}\right) v_5 + v_6\, \left(v_{10}\, v_{12} - v_{15}\, v_9\right)\right) \le 2\, v_{10}\, v_{12}\, v_{15}\, v_4 + v_{15}{}^2 v_5{}^2 + 2\, \left(v_{10}\, v_{18} - v_{12}\, v_{15}\right) v_5\, v_6 + v_{13}\, v_{18}\, v_4\, v_9 + v_8{}^2 \left(v_{10}{}^2 - v_{13}\, v_9\right) + v_6{}^2 \left(v_{12}{}^2 - v_{18}\, v_9\right) \wedge v_{12}{}^2 v_{16}\, v_4 + v_{11}{}^2 v_{18}\, v_4 + v_{16}\, v_{18}\, v_5{}^2 + v_{17}{}^2 v_4\, v_9 + 2\, v_8\, \left(\left(v_{11}\, v_{17} - v_{12}\, v_{16}\right) v_5 + v_7\, \left(v_{11}\, v_{12} - v_{17}\, v_9\right)\right) \le 2\, v_{11}\, v_{12}\, v_{17}\, v_4 + v_{17}{}^2 v_5{}^2 + 2\, \left(v_{11}\, v_{18} - v_{12}\, v_{17}\right) v_5\, v_7 + v_{16}\, v_{18}\, v_4\, v_9 + v_8{}^2 \left(v_{11}{}^2 - v_{16}\, v_9\right) + v_7{}^2 \left(v_{12}{}^2 - v_{18}\, v_9\right) \wedge v_{13}\, v_8{}^2 + v_{18}\, v_6{}^2 \le \left(v_{13}\, v_{18} - v_{15}{}^2\right) v_4 + 2\, v_{15}\, v_6\, v_8 \wedge v_{10}{}^2 v_{18} + v_{12}{}^2 v_{13} \le 2\, v_{10}\, v_{12}\, v_{15} + \left(v_{13}\, v_{18} - v_{15}{}^2\right) v_9 \wedge v_{15}{}^2 v_{16}\, v_4 + v_{13}\, v_{17}{}^2 v_4 + v_{14}{}^2 v_{18}\, v_4 + v_{16}\, v_{18}\, v_6{}^2 + 2\, \left(\left(v_{14}\, v_{17} - v_{15}\, v_{16}\right) v_6 + \left(-v_{13}\, v_{17} + v_{14}\, v_{15}\right) v_7\right) v_8 \le 2\, v_{14}\, v_{15}\, v_{17}\, v_4 + v_{13}\, v_{16}\, v_{18}\, v_4 +$$

$$
\begin{gathered}
{v_{17}}^2{v_6}^2 + 2\ (v_{14}\, v_{18} - v_{15}\, v_{17})\, v_6\, v_7 + \left(-v_{13}\, v_{18} + {v_{15}}^2\right) {v_7}^2 + \left(-v_{13}\, v_{16} + {v_{14}}^2\right) {v_8}^2 \land \\
2\, v_{12}\ (v_{11}\ (-v_{13}\, v_{17} + v_{14}\, v_{15}) + v_{10}\ (v_{14}\, v_{17} - v_{15}\, v_{16})) + {v_{10}}^2 v_{16}\, v_{18} + {v_{15}}^2 v_{16}\, v_9 + \\
v_{13}\, {v_{17}}^2 v_9 + {v_{14}}^2 v_{18}\, v_9 \leq {v_{12}}^2 \left(-v_{13}\, v_{16} + {v_{14}}^2\right) + {v_{10}}^2 {v_{17}}^2 + {v_{11}}^2 \left(-v_{13}\, v_{18} + {v_{15}}^2\right) + \\
2\, v_{10}\, v_{11}\ (v_{14}\, v_{18} - v_{15}\, v_{17}) + 2\, v_{14}\, v_{15}\, v_{17}\, v_9 + v_{13}\, v_{16}\, v_{18}\, v_9 \land v_{16}\, {v_8}^2 + v_{18}\, {v_7}^2 \leq \\
\left(v_{16}\, v_{18} - {v_{17}}^2\right) v_4 + 2\, v_{17}\, v_7\, v_8 \land {v_{11}}^2 v_{18} + {v_{12}}^2 v_{16} \leq 2\, v_{11}\, v_{12}\, v_{17} + \left(v_{16}\, v_{18} - {v_{17}}^2\right) v_9 \land \\
{v_{14}}^2 v_{18} + {v_{15}}^2 v_{16} \leq 2\, v_{14}\, v_{15}\, v_{17} + v_{13}\ \left(v_{16}\, v_{18} - {v_{17}}^2\right) \land 2\, v_{11}\, v_{12}\, v_{14}\, v_{15}\, v_4 + \\
{v_{12}}^2 v_{13}\, v_{16}\, v_4 + 2\, v_{10}\, v_{12}\, v_{14}\, v_{17}\, v_4 + 2\, v_{10}\, v_{11}\, v_{15}\, v_{17}\, v_4 + {v_{11}}^2 v_{13}\, v_{18}\, v_4 + {v_{10}}^2 v_{16}\, v_{18}\, v_4 + \\
2\, v_{14}\, v_{15}\, v_{17}\, {v_5}^2 + v_{13}\, v_{16}\, v_{18}\, {v_5}^2 + 2\, v_{12}\, v_{15}\, v_{16}\, v_5\, v_6 + 2\, v_{10}\, {v_{17}}^2 v_5\, v_6 + 2\, v_{11}\, v_{14}\, v_{18}\, v_5\, v_6 + \\
2\, v_{11}\, v_{12}\, v_{17}\, {v_6}^2 + {v_{15}}^2 v_{16}\, v_4\, v_9 + v_{13}\, {v_{17}}^2 v_4\, v_9 + {v_{14}}^2 v_{18}\, v_4\, v_9 + v_{16}\, v_{18}\, {v_6}^2 v_9 + \\
2\, v_8\ \left(\left(v_{12}\ \left(-v_{13}\, v_{16} + {v_{14}}^2\right) + v_{11}\ (v_{13}\, v_{17} - v_{14}\, v_{15}) + v_{10}\ (-v_{14}\, v_{17} + v_{15}\, v_{16})\right) v_5 + v_7\ (-v_{10}\, v_{12}\, v_{14} + v_{11}\ (- \right. \\
2\, v_7\ \left(\left(v_{12}\ (v_{13}\, v_{17} - v_{14}\, v_{15}) + v_{11}\ \left(-v_{13}\, v_{18} + {v_{15}}^2\right) + v_{10}\ (v_{14}\, v_{18} - v_{15}\, v_{17})\right) v_5 + v_6\ \left({v_{12}}^2 v_{14} - v_{12}\ (v_{10}\, v_{17} \right.\right. \\
{v_{12}}^2 {v_{14}}^2 v_4 + {v_{11}}^2 {v_{15}}^2 v_4 + 2\, v_{10}\, v_{12}\, v_{15}\, v_{16}\, v_4 + 2\, v_{11}\, v_{12}\, v_{13}\, v_{17}\, v_4 + {v_{10}}^2 {v_{17}}^2 v_4 + \\
2\, v_{10}\, v_{11}\, v_{14}\, v_{18}\, v_4 + {v_{15}}^2 v_{16}\, {v_5}^2 + v_{13}\, {v_{17}}^2 {v_5}^2 + {v_{14}}^2 v_{18}\, {v_5}^2 + 2\, v_{12}\, v_{14}\, v_{17}\, v_5\, v_6 + \\
2\, v_{11}\, v_{15}\, v_{17}\, v_5\, v_6 + 2\, v_{10}\, v_{16}\, v_{18}\, v_5\, v_6 + {v_{12}}^2 v_{16}\, {v_6}^2 + {v_{11}}^2 v_{18}\, {v_6}^2 + 2\, v_{14}\, v_{15}\, v_{17}\, v_4\, v_9 + \\
v_{13}\, v_{16}\, v_{18}\, v_4\, v_9 + {v_{17}}^2 {v_6}^2 v_9 + \\
{v_8}^2 \left({v_{11}}^2 v_{13} - 2\, v_{10}\, v_{11}\, v_{14} + {v_{10}}^2 v_{16} + \left(-v_{13}\, v_{16} + {v_{14}}^2\right) v_9\right) + \\
{v_7}^2 \left({v_{12}}^2 v_{13} - 2\, v_{10}\, v_{12}\, v_{15} + {v_{10}}^2 v_{18} + \left(-v_{13}\, v_{18} + {v_{15}}^2\right) v_9\right) \land v_7 < \\
v_2 \land v_1\ (v_{12} + v_3 - 2\, v_8) < 0
\end{gathered}
$$

Hypotheses:

$$v_{19} < 0$$

4.39 Constant absolute risk aversion with states delineated 1001

Assumptions:

$$
\begin{gathered}
v_3 < 1 \land (-1 + v_{27})^2\, v_{93} + (-1 + v_{28})^2\, v_{94} + (-1 + v_{29})^2\, v_{95} + (-1 + v_{30})^2\, v_{96} + \\
(-1 + v_{31})^2\, v_{97} + (-1 + v_{32})^2\, v_{98} + (-1 + v_{33})^2\, v_{99} + v_{100}\ (-1 + v_{34})^2 + \\
v_{101}\ (-1 + v_{35})^2 + (-1 + v_3)^2\, v_{69} + (-1 + v_4)^2\, v_{70} + (-1 + v_5)^2\, v_{71} + (v_6 - 1)^2\, v_{72} + \\
(v_7 - 1)^2\, v_{73} + (-1 + v_{10})^2\, v_{76} + (-1 + v_{11})^2\, v_{77} + (-1 + v_{12})^2\, v_{78} + (-1 + v_{13})^2\, v_{79} + \\
v_{74}\ (-1 + v_8)^2 + (-1 + v_{14})^2\, v_{80} + (-1 + v_{15})^2\, v_{81} + (-1 + v_{16})^2\, v_{82} + (-1 + v_{17})^2\, v_{83} + \\
(-1 + v_{18})^2\, v_{84} + (-1 + v_{19})^2\, v_{85} + (-1 + v_{20})^2\, v_{86} + (-1 + v_{21})^2\, v_{87} + (-1 + v_{22})^2\, v_{88} + \\
(-1 + v_{23})^2\, v_{89} + v_{75}\ (-1 + v_9)^2 + (-1 + v_{24})^2\, v_{90} + (-1 + v_{25})^2\, v_{91} + (-1 + v_{26})^2\, v_{92} < \\
0 \land 0 < v_1 \land 33 < v_3 + v_4 + v_7 + v_8 + v_6 + v_5 + v_9 + v_{10} + v_{11} + v_{12} + v_{21} + v_{19} + v_{22} + \\
v_{20} + v_{15} + v_{13} + v_{16} + v_{14} + v_{17} + v_{23} + v_{18} + v_{24} + v_{26} + v_{27} + v_{29} + v_{28} + v_{30} + v_{31} + \\
v_{25} + v_{34} + v_{32} + v_{35} + v_{33} \land (-1 + v_{26})\, v_{59} + v_{39}\ (v_6 - 1) + (-1 + v_{27})\, v_{60} + \\
(-1 + v_{28})\, v_{61} + (-1 + v_{29})\, v_{62} + (-1 + v_{30})\, v_{63} + (-1 + v_{31})\, v_{64} + (-1 + v_{32})\, v_{65} + \\
(-1 + v_{33})\, v_{66} + (-1 + v_{34})\, v_{67} + (-1 + v_{35})\, v_{68} + (-1 + v_{12})\, v_{45} + (-1 + v_{13})\, v_{46} + \\
(-1 + v_{14})\, v_{47} + (-1 + v_{15})\, v_{48} + (-1 + v_{16})\, v_{49} + v_{38}\ (-1 + v_5) + (-1 + v_{17})\, v_{50} + \\
(-1 + v_{18})\, v_{51} + (-1 + v_{19})\, v_{52} + (-1 + v_{20})\, v_{53} + (-1 + v_{21})\, v_{54} + (-1 + v_{22})\, v_{55} + \\
(-1 + v_{23})\, v_{56} + (-1 + v_{24})\, v_{57} + (-1 + v_{25})\, v_{58} + v_{42}\ (-1 + v_9) + (-1 + v_3)\, v_{36} + \\
v_{37}\ (-1 + v_4) + (-1 + v_{10})\, v_{43} + (-1 + v_{11})\, v_{44} + v_{40}\ (v_7 - 1) + v_{41}\ (-1 + v_8) = \\
0 \land v_{69} = -v_1\, v_{36} \land v_{70} = -v_1\, v_{37} \land v_{71} = -v_1\, v_{38} \land v_{72} = -v_1\, v_{39} \land v_{73} = -v_1\, v_{40} \land v_{74} = \\
-v_1\, v_{41} \land v_{75} = -v_1\, v_{42} \land v_{76} = -v_1\, v_{43} \land v_{77} = -v_1\, v_{44} \land v_{78} = -v_1\, v_{45} \land v_{79} =
\end{gathered}
$$

$$
\begin{gathered}
-v_1\,v_{46} \wedge v_{80} = -v_1\,v_{47} \wedge v_{81} = -v_1\,v_{48} \wedge v_{82} = -v_1\,v_{49} \wedge v_{83} = -v_1\,v_{50} \wedge v_{84} = \\
-v_1\,v_{51} \wedge v_{85} = -v_1\,v_{52} \wedge v_{86} = -v_1\,v_{53} \wedge v_{87} = -v_1\,v_{54} \wedge v_{88} = -v_1\,v_{55} \wedge v_{89} = \\
-v_1\,v_{56} \wedge v_{90} = -v_1\,v_{57} \wedge v_{91} = -v_1\,v_{58} \wedge v_{92} = -v_1\,v_{59} \wedge v_{93} = -v_1\,v_{60} \wedge v_{94} = \\
-v_1\,v_{61} \wedge v_{95} = -v_1\,v_{62} \wedge v_{96} = -v_1\,v_{63} \wedge v_{97} = -v_1\,v_{64} \wedge v_{98} = -v_1\,v_{65} \wedge v_{99} = \\
-v_1\,v_{66} \wedge v_{100} = -v_1\,v_{67} \wedge v_{101} = -v_1\,v_{68} \wedge (-1+v_{15})\,(v_{15}\,v_2 - v_2 + 1)\,v_{81} + \\
(-1+v_{16})\,(v_{16}\,v_2 - v_2 + 1)\,v_{82} + (-1+v_{17})\,(v_{17}\,v_2 - v_2 + 1)\,v_{83} + \\
(-1+v_{18})\,(v_{18}\,v_2 - v_2 + 1)\,v_{84} + (-1+v_{19})\,(v_{19}\,v_2 - v_2 + 1)\,v_{85} + \\
(-1+v_{20})\,(v_2\,v_{20} - v_2 + 1)\,v_{86} + (-1+v_{21})\,(v_2\,v_{21} - v_2 + 1)\,v_{87} + \\
(-1+v_{22})\,(v_2\,v_{22} - v_2 + 1)\,v_{88} + (-1+v_{23})\,(v_2\,v_{23} - v_2 + 1)\,v_{89} + \\
v_{75}\,(-1+v_9)\,(v_2\,v_9 - v_2 + 1) + (-1+v_{24})\,(v_2\,v_{24} - v_2 + 1)\,v_{90} + \\
v_{100}\,(-1+v_{34})\,(v_2\,v_{34} - v_2 + 1) + v_{101}\,(-1+v_{35})\,(v_2\,v_{35} - v_2 + 1) + \\
(-1+v_3)\,(v_2\,v_3 - v_2 + 1)\,v_{69} + (-1+v_4)\,(v_2\,v_4 - v_2 + 1)\,v_{70} + \\
(-1+v_5)\,(v_2\,v_5 - v_2 + 1)\,v_{71} + (v_6 - 1)\,(v_2\,v_6 - v_2 + 1)\,v_{72} + \\
(v_7 - 1)\,(v_2\,v_7 - v_2 + 1)\,v_{73} + (-1+v_{10})\,(v_{10}\,v_2 - v_2 + 1)\,v_{76} + \\
(-1+v_{11})\,(v_{11}\,v_2 - v_2 + 1)\,v_{77} + (-1+v_{12})\,(v_{12}\,v_2 - v_2 + 1)\,v_{78} + \\
(-1+v_{13})\,(v_{13}\,v_2 - v_2 + 1)\,v_{79} + v_{74}\,(-1+v_8)\,(v_2\,v_8 - v_2 + 1) + \\
(-1+v_{14})\,(v_{14}\,v_2 - v_2 + 1)\,v_{80} + (-1+v_{25})\,(v_2\,v_{25} - v_2 + 1)\,v_{91} + \\
(-1+v_{26})\,(v_2\,v_{26} - v_2 + 1)\,v_{92} + (-1+v_{27})\,(v_2\,v_{27} - v_2 + 1)\,v_{93} + \\
(-1+v_{28})\,(v_2\,v_{28} - v_2 + 1)\,v_{94} + (-1+v_{29})\,(v_2\,v_{29} - v_2 + 1)\,v_{95} + \\
(-1+v_{30})\,(v_2\,v_{30} - v_2 + 1)\,v_{96} + (-1+v_{31})\,(v_2\,v_{31} - v_2 + 1)\,v_{97} + \\
(-1+v_{32})\,(v_2\,v_{32} - v_2 + 1)\,v_{98} + (-1+v_{33})\,(v_2\,v_{33} - v_2 + 1)\,v_{99} = 0
\end{gathered}
$$

Hypotheses:

$$v_2 = 0$$

4.40 Supply-Demand: Incidence parameter for scenario analysis 0015

Assumptions:

$$
\begin{gathered}
v_2 = 0 \wedge v_4 = v_3 \wedge v_5 = 0 \wedge v_9 = 1 \wedge v_{10} = 1 \wedge v_1\,v_9 + v_{11}\,v_5 = v_7 \wedge v_{11}\,v_6 + v_2\,v_9 = \\
v_8 \wedge v_{10}\,v_3 + v_{12}\,v_5 = v_7 \wedge v_{10}\,v_4 + v_{12}\,v_6 = v_8 \wedge 0 < v_{12} \wedge v_7 < 0 \wedge v_{11} < 0
\end{gathered}
$$

Hypotheses:

$$v_{11}\,v_7 - v_{11}\,v_8 + v_{12}\,v_8 = 0 \wedge v_{11} - v_{12} \neq 0 \wedge -v_{11}\,v_7 + v_{11}\,v_8 - v_{12}\,v_8 = 0 \wedge v_{11} - v_{12} \neq 0$$

4.41 NGM: Signing the impact of purchases on labor, separable case 0029

Assumptions:

$$
\begin{gathered}
v_8 = 0 \wedge v_{11}\,v_7 + v_{12}\,v_6 = 1 + v_5 \wedge v_{13}\,v_3\,v_4\,v_7 + v_{11}\,v_3\,v_9 + v_{11}\,v_4\,v_7 = 1 \wedge v_{14}\,v_6 = \\
v_8 \wedge v_{13}\,{v_{17}}^2 v_4\,v_7 + v_{11}\,{v_{17}}^2 v_9 - v_{13}\,{v_{17}}^2 v_7 + v_{15}\,v_{18}\,v_7 + v_{15}\,v_{19}\,v_5 - v_{16}\,v_{17}\,v_7 - v_{17}\,v_{18}\,v_5 = \\
0 \wedge {v_{17}}^2 \neq 0 \wedge 0 < v_1 \wedge 0 < v_3 \wedge 0 < v_9 \wedge 0 < v_{10} \wedge 0 < v_{11} \wedge 0 < v_{12} \wedge 0 < \\
v_{13}\,v_3 + v_{11} \wedge 0 < -\left(-v_{15}\,v_{18} + v_{16}\,v_{17}\right) {v_{17}}^2 \wedge 0 < -\left(-v_{15}\,v_{19} + v_{17}\,v_{18}\right) {v_{17}}^2 \wedge 0 \leq \\
v_2 \wedge v_4 < 1 \wedge v_{14} < 0 \wedge 0 \leq v_4 \wedge v_{13} \leq 0
\end{gathered}
$$

Hypotheses:

$$v_7 = 0 \wedge -v_{11}\, v_{17}{}^2 v_9 + v_{15}\, v_{19} - v_{17}\, v_{18} = 0 \wedge v_{17}{}^2 \neq 0 \vee 0 < -v_7\, \left(v_{11}\, v_{17}{}^2 v_9 - v_{15}\, v_{19} + v_{17}\, v_{18}\right) v_{17}{}^2$$

4.42 Vector mode: Correlation between wealth and marginal utility 0085

Assumptions:

$$-v_3\, v_1 + v_4 = 0 \wedge v_3 \neq 0 \wedge -v_2\, v_3 + v_5 = 0 \wedge v_3 \neq 0 \wedge v_5 - v_7 = 0 \wedge v_3 \neq 0 \wedge 0 < -\left(v_3 - v_4\right) v_3 \wedge 0 < v_3\, v_5 \wedge 1 \leq v_3 \wedge 0 \leq v_6 \wedge v_4{}^2 \leq v_3\, v_6 \wedge 0 \leq v_8 \wedge v_5{}^2 \leq v_3\, v_8 \wedge v_7{}^2 \leq v_6\, v_8$$

Hypotheses:

$$v_1\, v_2\, v_3 + v_7 < v_1\, v_5 + v_2\, v_4$$

4.43 Industry Equilibrium: Determinants of long-run capital-labor complementarity 0066

Assumptions:

$$5 + v_6 = 1 \wedge v_{12} = v_{10} + v_5\, \left(-v_{10} + v_9\right) \wedge v_{11}\, v_{13}\, v_6 = v_{12} \wedge v_{11}\, \left(v_{13}\, v_6 - v_{14}\, v_5\right) = v_{10} \wedge 0 < v_1 \wedge 0 < v_2 \wedge 0 < v_3 \wedge 0 < v_4 \wedge 0 < v_7 \wedge 0 < v_8 \wedge 0 < v_{16} \wedge 0 < -\left(-v_1\, v_{15} + v_{16}\, v_2\right) v_1 \wedge 0 < v_6 \wedge v_6 < 1$$

Hypotheses:

$$v_{11}\, \left(v_{13} + v_{14}\right) v_6 = v_9$$

4.44 Industry Equilibrium: Determinants of short-run capital-labor complementarity 0067

Assumptions:

$$v_5 + v_6 = 1 \wedge -v_3\, v_5\, v_8 + v_1\, v_4 = 0 \wedge v_3\, v_8 \neq 0 \wedge -v_3\, v_6\, v_8 + v_2\, v_7 = 0 \wedge v_3\, v_8 \neq 0 \wedge v_9 = 0 \wedge v_{12}\, v_{16} - v_{13}\, v_{16} - v_{10} + v_9 = 0 \wedge v_{12} - v_{13} \neq 0 \wedge v_{18} = v_8 \wedge -v_1\, v_9 + v_{19} = 0 \wedge v_1 \neq 0 \wedge -v_{10}\, v_2 + v_{20} = 0 \wedge v_2 \neq 0 \wedge -v_{11}\, v_3 + v_{21} = 0 \wedge v_3 \neq 0 \wedge -v_{12}\, v_4 + v_{22} = 0 \wedge v_4 \neq 0 \wedge -v_{13}\, v_7 + v_{23} = 0 \wedge v_7 \neq 0 \wedge -v_{14}\, v_8 + v_{24} = 0 \wedge v_8 \neq 0 \wedge v_{27}\, v_8 = v_1 \wedge v_{17}\, v_7\, v_8 - v_{27}\, v_4\, v_8 - v_2\, v_7 = 0 \wedge v_7 \neq 0 \wedge v_{17}\, v_{23}\, v_7 + v_{22}\, v_{27}\, v_7 - v_{23}\, v_{27}\, v_4 - v_{21}\, v_7 = 0 \wedge v_7 \neq 0 \wedge -v_{15}\, v_{18} + v_{28}\, v_3 = 0 \wedge v_{18} \neq 0 \wedge v_{21}\, v_{28} = v_{24} \wedge v_1\, v_{19}\, v_{25} + v_1\, v_{20}\, v_{29} - v_{19}\, v_2\, v_{29} - v_1\, v_{24} = 0 \wedge v_1 \neq 0 \wedge v_{22}\, v_{31}\, v_7\, v_8 - v_{23}\, v_{31}\, v_4\, v_8 + v_{24}\, v_{27}\, v_7{}^2 - v_{19}\, v_7{}^2 = 0 \wedge v_7{}^2 \neq 0 \wedge v_{17}\, v_{24}\, v_7{}^3 - v_{22}\, v_{31}\, v_4\, v_7\, v_8 + v_{23}\, v_{31}\, v_4{}^2 v_8 - v_{24}\, v_{27}\, v_4\, v_7{}^2 - v_{20}\, v_7{}^3 = 0 \wedge v_7{}^3 \neq 0 \wedge 0 < v_1 \wedge 0 < v_2 \wedge 0 < v_3 \wedge 0 < v_4 \wedge 0 < v_7 \wedge 0 < v_8 \wedge 0 < v_{30} \wedge 0 < -\left(-v_1\, v_{26} + v_2\, v_{30}\right) v_1 \wedge 0 < v_6 \wedge v_6 < 1$$

Hypotheses:

$$v_{12}\, \left(-v_{15}\, v_5 + v_{16}\, v_6\right) = v_{13}\, \left(v_{15} + v_{16}\right) v_6$$

4.45 Industry Equilibrium: Long-run relationship between real output and real wages 0062

Assumptions:

$$-v_3\,v_5\,v_8 + v_1\,v_4 = 0 \land v_3\,v_8 \neq 0 \land -v_3\,v_6\,v_8 + v_2\,v_7 = 0 \land v_3\,v_8 \neq 0 \land v_{12} = 0 \land v_{17} = v_8 \land -v_1\,v_9 + v_{18} = 0 \land v_1 \neq 0 \land -v_{10}\,v_2 + v_{19} = 0 \land v_2 \neq 0 \land -v_{11}\,v_3 + v_{20} = 0 \land v_3 \neq 0 \land -v_{12}\,v_4 + v_{21} = 0 \land v_4 \neq 0 \land -v_{13}\,v_7 + v_{22} = 0 \land v_7 \neq 0 \land -v_{14}\,v_8 + v_{23} = 0 \land v_8 \neq 0 \land v_{24}\,v_8 = v_1 \land v_{16}\,v_7\,v_8 - v_{24}\,v_4\,v_8 - v_2\,v_7 = 0 \land v_7 \neq 0 \land v_{16}\,v_{22}\,v_7 + v_{21}\,v_{24}\,v_7 - v_{22}\,v_{24}\,v_4 - v_{20}\,v_7 = 0 \land v_7 \neq 0 \land -v_{15}\,v_{17} + v_{25}\,v_3 = 0 \land v_{17} \neq 0 \land v_{20}\,v_{25} = v_{23}$$

Hypotheses:

$$v_{13}\,v_{15}\,v_6 = v_{14}$$

4.46 Supply-Demand: Price and quantity impact together reveal quantity determinants 0006

Assumptions:

$$v_5 = 1 \land v_6 = 1 \land v_1\,v_5 + v_3\,v_7 = v_4 \land v_2\,v_6 + v_3\,v_8 = v_4 \land 0 < v_8 \land v_7 < 0$$

Hypotheses:

$$0 < v_1 \lor v_3 \leq 0 \lor v_4 \leq 0$$

4.47 NGM (Neoclassical Growth Model): Labor taxes reduce steady-state labor, capital, consumption 0021

Assumptions:

$$v_7 = 0 \land v_8 = 0 \land v_{21} - v_5 = v_4 \land v_{21}\,v_7 - v_{21} + v_5 + v_6 = 0 \land v_7 - 1 \neq 0 \land v_{19}\,v_3 + v_2\,v_{21} = v_{12} \land v_{10}\,v_{21} - v_{10}\,v_5 + v_{11}\,v_{19} - v_9 = 0 \land -v_2\,v_{22} - v_{20}\,v_3 = 0 \land v_3 \neq 0 \land -v_{10}\,{v_{15}}^2 v_{22}\,v_8 - v_{11}\,{v_{15}}^2 v_{20}\,v_8 + v_{10}\,{v_{15}}^2 v_{22} + v_{11}\,{v_{15}}^2 v_{20} - v_{11}\,v_{13}\,v_{16} + v_{11}\,v_{14}\,v_{15} - v_{13}\,v_{17}\,v_9 - {v_{15}}^2 v_{19} + v_{15}\,v_{16}\,v_9 = 0 \land {v_{15}}^2 \neq 0 \land v_2\,v_{23} - v_{22}\,v_3 = 0 \land v_2 \neq 0 \land v_{10}\,v_{23} + v_{11}\,v_{22} = 0 \land 0 < v_1 \land 0 < v_2 \land 0 < v_3 \land 0 < v_6 \land 0 < v_{15} \land 0 < -\left(-v_{13}\,v_{16} + v_{14}\,v_{15}\right) {v_{15}}^2 \land 0 < -\left(-v_{13}\,v_{17} + v_{15}\,v_{16}\right) {v_{15}}^2 \land 0 < v_{18} \land 0 < v_{19} \land 0 < v_{21} \land 0 \leq v_4 \land 0 \leq v_5 \land 0 \leq v_{22} \land v_7 < 1 \land v_8 < 1 \land v_{13} < 0 \land v_{14} < 0 \land v_{17} < 0 \land v_{20} < 0 \land v_{14} + v_{16}\,v_{19}\,\left(1 - v_8\right) + v_{15}\,v_{20}\,\left(1 - v_8\right) < 0 \land v_{23} < 0$$

Hypotheses:

$$\mathit{v9} < 0 \land \mathit{v10} < 0 \land \mathit{v11} < 0$$

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